

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN



Fall term 2022

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## Selected Topics in Complex Geometry

Sheet 07

**Exercise 1.** An *n*-grading on a complex vector space *H* is a decomposition  $H = \bigoplus_{p \in \mathbb{Z}^n} H^p$ . Show that this datum is equivalent to the datum of a holomorphic representation of  $(\mathbb{C}^{\times})^n$  on *H*. Use this to prove that there is an equivalence of categories between *n*-graded vector spaces and representations (which classes of maps can you consider?)

**Exercise 2.** Let  $A = \bigoplus A^k$  be a cohomology algebra with Hodge structure. Use Deligne's Lemma to prove that the following sets generate (rational) sub Hodge structures of  $A^k_{\mathbb{C}}$ :

 $Z_1 := \{ \alpha \in A^k_{\mathbb{C}} \mid \alpha^2 = 0 \} \qquad Z_2 := \left\{ \alpha \in A^k_{\mathbb{C}} \mid \operatorname{rk} \left( A^l_{\mathbb{C}} \xrightarrow{\alpha} A^{l+k}_{\mathbb{C}} \right) \le m \right\}$ 

**Exercise 3.** Show that every pure  $\mathbb{Q}$ -Hodge structure of dimension 2 admits a polarisation. Show that this is not true if the dimension is 4.

Hand-in: Via Email or in person to Jonas Stelzig until We, 15.06., 14:00.