Fall term 2022

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Selected Topics in Complex Geometry

Sheet 05

Exercise 1. Show in two ways that the product of two projective manifolds is again projective:

1. using the Kodaira embedding theorem,
2. by means of an explicit embedding of $\mathbb{C}\mathrm{P}^n \times \mathbb{C}\mathrm{P}^m \to \mathbb{C}\mathrm{P}^N$.

Exercise 2. Let $F := F_m^n \subseteq \mathbb{C}\mathrm{P}^n$ be the Fermat hypersurface, i.e. the zero locus of $z_0^m + ... + z_n^m$. Show that F_m^n is a complex manifold and compute the dimensions $H^0(F, K_F^{\otimes k})$ for all k.

Exercise 3. Show that any vector bundle E on a projective manifold X admits a surjection $(L^{\otimes k})^{\oplus l} \to E$ with L an ample line bundle and some $k < 0 < l$. (Consider sheaves of homomorphisms and mimick the argument in the proof of Kodaira-embedding)

Exercise 4. Show that a compact complex torus V/Γ is projective if and only if there exists an alternating bilinear form

$$\omega : V \times V \to \mathbb{R}$$

such that

1. $\omega(iv, iw) = \omega(v, w)$ for all $v, w \in V$,
2. $\omega(\_, i\_)$ is positive definite,
3. $\omega(v, w) \in \mathbb{Z}$ for all $v, w \in \Gamma$.

Hand-in: Via Email or in person to Jonas Stelzig until Mo, 30.05., 14:00.