Bol. Soc. Mat. Mexicana (3) Vol. 2, 1996

LOCAL PROPERTY OF VISCOSITY SOLUTIONS OF FULLY NONLINEAR SECOND ORDER ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

MYTHILY RAMASWAMY AND S. RAMASWAMY

Abstract

The two approaches to viscosity solutions of fully nonlinear elliptic pde's, one as introduced in Caffarelli and Wang and the other as in Crandall, Ishii and Lions, are studied. For some class of uniformly elliptic operators, the first notion is shown to be local and is equivalent to the second one.

1. Introduction

In this note, we study the two approaches to the viscosity solution of fully nonlinear second order elliptic pde's: one approach, similar to that developed in Caffarelli and Wang ([1], [5]) and the other one, as in Crandall, Ishii and Lions [2]. We show that the first notion is local and is equivalent to the second one for some class of uniformly elliptic operators, thus establishing a link between the two approaches.

2. Viscosity solutions

Let Ω be a nonempty open bounded subset of $\mathbb{R}^n$ and S_n be the subspace of all symmetric $n \times n$ matrices. Let F be a continuous map from $S_n \times \mathbb{R}^n \times \mathbb{R} \times \Omega$ to $\mathbb{R}$, satisfying the uniform ellipticity condition:

$$\Lambda\|M\| \geq F(N+M, p, z, x) - F(N, p, z, x) \geq \lambda\|M\|, \tag{2.1}$$

for all $N \in S_n$, $M \geq 0$ and $(p, z, x) \in \mathbb{R}^n \times \mathbb{R} \times \Omega$. Here $\|M\| = (\sum_j \sum_i |m_{ij}|^2)^{1/2}$ for the matrix $M = [m_{ij}]$. First we study the viscosity solutions of

$$F(D^2u(x), Du(x), u(x), x) = 0 \tag{2.2}$$

in the spirit of [1] and [5].

1991 *Mathematics Subject Classification.* 35B99, 35D99.
Keywords and phrases: Fully nonlinear elliptic equations, viscosity super and subsolutions.