MATHEMATISCHES INSTITUT
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PDG I
(Tutorium)

Tutorial 5

(Homogeneous harmonic polynomials in $\mathbb{R}^n$)

In this tutorial we covered the following topics:

- Considering the Laplace operator as a linear map from $\mathcal{P}_m(\mathbb{R}^n)$ to $\mathcal{P}_{m-2}(\mathbb{R}^n)$, with kernel $\mathcal{H}_m(\mathbb{R}^n)$
- Using properties of homogeneous functions from Tutorial 4 to show that there are no harmonic polynomials of the form $|x|^2 p(x)$, where $p \in \mathcal{P}_m(\mathbb{R}^n)$. Using elementary results from linear algebra to deduce the dimension of $\mathcal{H}_m(\mathbb{R}^n)$.
- Decomposing an m-homogeneous polynomial in terms of harmonic polynomials. That is, every $p \in \mathcal{P}_m(\mathbb{R}^n)$ has a unique decomposition of the form

$$p(x) = h_m(x) + |x|^2 h_{m-2}(x) + |x|^4 h_{m-4}(x) + \ldots + |x|^{2k} h_{m-2k}(x)$$

  where $k = \lceil \frac{m}{2} \rceil$ and $h_j \in \mathcal{H}_j(\mathbb{R}^n)$.