MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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WS 2014/15
– Blatt 3 –
29.10.2014

PDG I
(Zentralübung)

Problem Sheet 3

Question 1

Modify the proof of the mean-value formulas to show for $n \geq 3$ that

$$u(0) = ⨏_{\partial B(0,r)} g \, \mathrm{d}S + \frac{1}{n(n-2)\alpha(n)} \int_{B(0,r)} \left(\frac{1}{|x|^{n-2}} - \frac{1}{r^{n-2}} \right) f \, \mathrm{d}x \,,$$

provided

$$\begin{cases} -\Delta u = f & \text{in } B(0,r) \\ u = g & \text{on } \partial B(0,r) \,. \end{cases}$$

Question 2

Suppose $u \in C^\infty(\mathbb{R}^n)$ satisfies $\Delta u = 0$. Let O be an orthogonal $n \times n$ matrix, and define

$$v(x) := u(Ox) \,, \qquad (x \in \mathbb{R}^n) \,.$$

Show that $\Delta v = 0$ too (so Laplace's equation is rotation invariant).

Question 3

Suppose $u \colon x \in \mathbb{R}^n \setminus \{0\} \to \mathbb{R}$ has the form $u(x) = f(|x|)$, where $f \in C^2(0,\infty)$ (so u is rotationally invariant). Show that

$$\Delta u(x) = f''(|x|) + \frac{n-1}{|x|} f'(|x|) \,.$$

Use this to determine all rotationally invariant *harmonic* maps u on $\mathbb{R}^n \setminus \{0\}$.

Deadline for handing in: 0800 Wednesday 5 November

Please put solutions in Box 17, 1st floor (near the library)

Homepage: `http://www.mathematik.uni-muenchen.de/~soneji/pde1.php`