



Prof. Dr. Bachmann
A. Dietlein, R. Schulte

PARTIAL DIFFERENTIAL EQUATIONS I
HOMEWORK SHEET 13

WS 2016/17
January 23, 2017

Exercise 1 (5 Points). Let $g \in C^1(\mathbb{R})$. Use the method of characteristics to find a solution of the initial value problem

$$\begin{cases} u_{x_1}(x_1, x_2) + u_{x_2}(x_1, x_2) = u^2(x_1, x_2) & \text{for } (x_1, x_2) \in \mathbb{R} \times (0, \infty), \\ u(x_1, 0) = g(x_1) & \text{for } x_1 \in \mathbb{R}. \end{cases}$$

Exercise 2 (5 Points). Let $g \in C^1(\mathbb{R})$. Use the method of characteristics to find a solution of the initial value problem

$$\begin{cases} x_1^2 u_{x_1}(x_1, x_2) + x_2^2 u_{x_2}(x_1, x_2) = u^2(x_1, x_2) & \text{for } (x_1, x_2) \in \mathbb{R}^2, \\ u(x, 2x) = x^2 & \text{for } x \in \mathbb{R}. \end{cases}$$

Exercise 3 (5 Points). Use the method of characteristics to find all solutions of

$$u_{x_1}(x_1, x_2) + x_1^2 u_{x_2}(x_1, x_2) = 1 \quad \text{for } (x_1, x_2) \in \mathbb{R}^2$$

Exercise 4 (5 Points). Use the method of characteristics to find a solution of

$$\begin{cases} u(x_1, x_2) u_{x_1}(x_1, x_2) + u_{x_2}(x_1, x_2) = 1 & \text{for } (x_1, x_2) \in (0, 1)^2, \\ u(x, x) = \frac{x}{2} & \text{for } x \in (0, 1). \end{cases}$$

You can drop your homework solutions until **Monday, January 30** at **16 o'clock** into the appropriate letterbox on the first floor near the library.