Let G be a Lie group with a biinvariant metric h, i.e. a metric on TG which is right and left invariant.
Prove the following assertions:

a) For the Levi-Civita connection corresponding to h

$$\nabla_X Y = \tfrac{1}{2}[X,Y] \,,$$

$$F(X,Y)Z = -\tfrac{1}{4}[[X,Y],Z] \,,$$

$$\mathrm{Ric}(X,Y) = -\tfrac{1}{4}\kappa(X,Y) \,, \qquad X,Y,Z \in \mathfrak{g} = \mathrm{Lie}\, G.$$

Here, is κ the Killing form $\kappa(X,Y) = \mathrm{Tr}(\mathrm{ad}(X)\circ \mathrm{ad}(Y))$.

b) The geodesics are the solutions of the left invariant (right?) vector fields.

c) At every point $a \in G$ there is an isometry s (i.e. $s : G \to G$ is a diffeomorphism, s.th. $T_g s : T_g G \to T_{s(g)} G$ is an isometry for each $g \in G$) with $s(a) = a$ and $T_a s = -\mathrm{id}_{T_a G}$. (Such an isometry of mfd with metric is called a symmetry.)

d) In case of $G = SU(2)$ the scalar product

$$\langle X, Y \rangle = -\tfrac{1}{2}\mathrm{Tr}(X \circ Y) \,, \quad X, Y \in \mathrm{Lie}\, G = su(2)$$

defines a biinvariant metric h on such that (G,h) is isometric to S^3 with the induced metric $S^3 \subset \mathbb{R}^4$.