Let $\pi_E : E \to M$ be a vector bundle on the mfd M, and let $Q : V_E \to E$ the "strange" projection of the vertical bundle $V_E \xrightarrow[\tau_E]{} E$, $V_E := \ker T\pi_E$: For $\xi \in E_a$, Q_ξ is the inverse of $R_\xi : E_a \to V_{E,\xi}$, $R_\xi(\eta) := [\xi + t\eta]_\xi$.

a) Describe Q and τ_E with respect to a bundle chart, and compare the two projections.

b) The $(R_\xi)_{\xi \in E}$ induce an isomorphism $\pi_E^* E \xrightarrow{R} V_E$ of vector bundes $(\xi, \eta) \longmapsto [\xi + t\eta]_\xi$ with $pr_1 = \tau_E \circ R$ and Q corresponding to the projection $pr_2 : \pi^* E \to E$.

c) Complete the proof of theorem 24.12 by first establishing the Leibniz rule

$$D(fs) = df\, s + f\, Ds \qquad \text{for } f \in \mathcal{E}(W),\ s \in \Gamma(W, E)$$

where $D = D_C$ is induced by a horizontal connection $C : \pi^* TM \to TE : D_C s(X) := Q \circ v \circ Ts(X)$, $X \in \mathcal{V}(W)$, and then showing $C = C_{D_C}$ and $D = D_{C_D}$. Here, C_D is the horizontal connection induced by a connection D (in the sense of 23.1) as in 24.8: $C(\xi, X) := [\eta]_\xi$ for $(\xi, X) \in \pi^* TM$, where η is the unique horizontal lift of a curve γ representing X $(X = [\gamma]_a,\ a = \tau_M(X) = \pi_E(\xi))$ through $\xi \in E_a$ (cf. 23.14).