Aufgabe 13

25.1.11

Mathematical Gauge Theory WS 10/11

1. Let $\xi = (P, \pi, M, G)$ be a pfb with connection H. Give a detailed proof of: „Parallel transport in P is independent of curves" $\iff$ There exists an isomorphism $\varphi : P \to M \times G$ of pfbs such that $T\varphi(H)$ is the canonical flat connection.

2. Show that there is a natural bijection between the space of all connections on a given vector bundle E and the space of the connections on the frame bundle $GL(E)$.

3. Let $\xi = (P, \pi, M, G)$ be a pfb with connection ω and let $\sigma \in \Gamma(M, AdP)$. We know that $\omega + \alpha$ is a connection (cf. 38.6). Let $\rho : G \to GL(r, \mathbb{K}^r)$ be a representation and let $E = E_\rho$ be the associated vector bundle (cf. section 35). Denote the associated connections on E with d^ω and $d^{\omega+\alpha}$ respectively (cf. 27.7 ff. and 38.2 / section 37). Show in detail

   1° $d^{\omega+\alpha}\eta = d^\omega \eta + \rho_*(\alpha)\eta \quad , \quad \eta \in \mathcal{A}^k(M, E)$

   2° $F^{\omega+\alpha} = F^\omega + d^\omega \alpha + \frac{1}{2}[\alpha, \alpha]$ for the respective curvatures $\in \mathcal{A}^2(M, E)$.