

Exercises on Mathematical Statistical Physics II Sheet 9

Problem 1 (Markov's Inequality with the Sixth Moment)

Show that for any Lipschitz continuous function f

$$\mathbb{P} \left(\left| \frac{1}{N-1} \sum_{j=2}^N f(x_1 - x_j) - f \star_q \rho(x_1) \right|_{\infty} \geq \varepsilon \right) \leq \frac{C}{N^3 \varepsilon^6}.$$

Use this estimate to get a better control of

$$\mathbb{E}(|X^t - \bar{X}^t|_{\infty})$$

Problem 2 (Two Boundary Conditions)

Let $J_t := \min\{1, \max\{f(t), g(t)\}\}$. Consider the time derivative of $\mathbb{E}(J_t)$. What boundary conditions do you get for seeking a Grönwall type estimate (see class)?

Problem 3 (Expectation Value - Probability)

Show, that for any random variable X with $\mathbb{E}(|X|) < \infty$

$$\mathbb{E}(|X|) = \int_0^{\infty} \mathbb{P}(|x| > a) da$$

The solutions to these exercises will be discussed on Friday, 27.01.2017.