LMU Munich
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Exercises on Mathematical Statistical Physics II
Sheet 7

Problem 1 (Scattering length)
Let $V \in L^\infty$ be a potential with compact support. Let, for some $\varepsilon > 0$, $W_N(x)$ be the potential given by $W_N(x) = C$ if $x < N^{-\varepsilon}$, $W_N(x) = 0$ else with C such that the scattering length of $V - W_N$ is zero. Let a be the scattering length of V.
Show that $\lim_{N\to\infty} \|W_N\|_1 = a$.

Problem 2 ($\beta \notin [0,1]$)
Let $H = -\sum_{j=1}^N \Delta_j + \sum_{j<k} N^{-1} V_N(x_j - x_K)$ where $V_N(x) := N^{3\beta} V(N^\beta x)$ with $\beta \notin [0,1]$. Argue that on any compact time interval the reduced density of Ψ evolves like a free state.

Problem 3 ($\beta = 1$)
Estimate the term

$$N^2 \left| \langle \Psi, q_1 q_2 \left(\nabla g(x_1 - x_2) \right) \nabla p_1 p_2 (\widehat{m} - \widehat{m}_2) \Psi \rangle \right|$$

with the g function defined in class.
Also try to estimate

$$N^2 \left| \langle \Psi, p_1 p_2 \left(\nabla g(x_1 - x_2) \right) \nabla q_1 q_2 (\widehat{m} - \widehat{m}_2) \Psi \rangle \right|$$

Why is this not working? What does that mean for the ansatz of the α in the $\beta = 1$ case?

The solutions to these exercises will be discussed on Friday, 23.12.2016.