LMU Munich
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Exercises on Mathematical Statistical Physics II
Sheet 6

Problem 1 (Hartree Energy Functional)
Let ϕ_t be a solution of the Hartree equation

$$i\frac{d}{dt}\phi_t = \left(-\Delta + V \star |\phi_t|^2\right)\phi_t \ .$$

Show that the energy given by

$$E(\phi) = \left\langle \phi_t, \left(-\Delta + \frac{1}{2}V \star |\phi_t|^2\right)\phi_t \right\rangle$$

is conserved under time evolution.

Problem 2 (Time Dependent External Field)

Let Ψ_t be solution of the Schrödinger equation with Hamiltonian

$$H := \sum_{j=1}^{N} -\Delta_j + A_t(x_j) + \sum_{j<k} V_N(x_j - x_k)$$

and ϕ_t be solution of the Hartree equation

$$i\frac{d}{dt}\phi_t = \left(-\Delta + A_t + V \star |\phi_t|^2\right)\phi_t \ .$$

Asume that the time dependent external field satisfies $\|\frac{d}{dt}A_t\|_\infty \leq C$ for some constant C uniform in time.
Estimate the energy difference

$$\Delta E(t) := \left| \frac{1}{N}\langle \Psi_t, H\Psi_t\rangle - \left\langle \phi_t, \left(-\Delta + A_t + \frac{1}{2}V \star |\phi_t|^2\right)\phi_t \right\rangle \right|$$

in terms of $\Delta E(0)$ and α with the smallest possible weight.

Problem 3 (Large Volume Revisited)
Let ρ, Λ be as in problem 3 on sheet 4. Show that $|d_t\alpha| \leq \alpha + \frac{C\Lambda^2}{\rho^2}$ using the weight function

$$m = \begin{cases} \frac{k^2}{\rho^2} & \forall\, k \leq \rho \\ 1 & \text{else} \end{cases}.$$

The solutions to these exercises will be discussed on Friday, 09.12.2016.