LMU Munich
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Exercises on Mathematical Statistical Physics II
Sheet 11

Problem 1 (Heat Equation from Brownian Motion)

Derive the heat equation $d_t\psi(x,t) = C\Delta\psi(x,t)$ as an effective equation for a system evolving due to Brownian motion (resp. Wiener process).

Problem 2 (Minimal Coupling)

Consider the N-particle bosonic Schrödinger equation with some weakly time-depentent perturbation of the Laplacian

$$i\frac{d}{dt}\Psi_t = \left(-\sum_{j=1}^{N}(i\nabla_j + A(t))^2 + \frac{1}{N}\sum_{k<j} V(|x_j - x_k|)\right)\Psi_t$$

with $\|A(t)\|_\infty < C < \infty, \|d_t A(t)\| < C < \infty$, $V \in L_r$ for $2 \leq r \leq \infty$ and $\Psi_0 = \prod_{j=1}^{N}\phi(x_j)$.
Show that the system is effectively described by a Hartree equation. At which places do you use that the wave function is symmetric?
What did you assume for the solution of the effective equation dependent on r?

Problem 3 (Vlasov equation)

Consider the Newtonian system we discussed in class,
i.e. $X = (Q,P) = (q_1, q_2, \ldots, q_N, p_1, \ldots, p_N) \in \mathbb{R}^{6N}$ with

$$\dot{Q} = P \qquad \dot{p}_j = N^{-1}\sum_{k\neq j} f(q_j - q_k)$$

Assume that $f(q) = \frac{q}{|q|^2}$ with cutoff at $N^{-\delta}$. Proof convergence of X against the auxiliary system $\overline{X}$ in probability for δ as large as possible.

The solutions to these exercises will be discussed on Friday, 10.02.2017.