LMU Munich
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Exercises on Mathematical Statistical Physics II
Sheet 10

Problem 1

Let $X^t = (A^t, B^t, C^t)$ and $\overline{X}^t = (\overline{A}^t, \overline{B}^t, \overline{C}^t)$ for some functions $A, B, C, \overline{A}, \overline{B}, \overline{C} : \mathbb{R}_0^+ \to V$, where V is some vector space with norm $\| \cdot \|$.
Assume that $A^0 = \overline{A}^0$, $B^0 = \overline{B}^0$ and $C^0 = \overline{C}^0$ as well as $\frac{d}{dt}\|A^t - \overline{A}^t\| \leq C_1 \|B^t - \overline{B}^t\|$, $\frac{d}{dt}\|B^t - \overline{B}^t\| \leq C_1 \|C^t - \overline{C}^t\|$ and $\frac{d}{dt}\|C^t - \overline{C}^t\| \leq C_1 (\ln N)^2 \|A^t - \overline{A}^t\| + N^{-\eta}$ for some constants $C_1, C_2, C_3 < \infty$ and some $\eta > 0$.
Show that for any $t > 0$ $\lim_{N\to\infty} \|A^t - \overline{A}^t\| + \|B^t - \overline{B}^t\| + \|C^t - \overline{C}^t\| = 0$.

Problem 2 (Coulomb Case with Cutoff)

Show for the Coulomb case with cutoff at $N^{-1/3+\delta}$ for some $\delta > 0$ that there is a $\eta > 0$ such that

$$\mathbb{P}\left(\left\| X^t - \overline{X}^t \right\|_\infty \geq N^{-1/3+\delta} \right) \leq C N^{-\eta} \, .$$

Problem 3 (Bounded Lipschitz Norm)

Show explicitly that the bounded Lipschitz distance

$$d_{BL}(\rho, \sigma) := \sup_{\|f\|_L \leq 1} \left| \int_{\mathbb{R}^n} (\rho - \sigma) f(x) d^n x \right|$$

is indeed a metric on $\mathbb{R}^n$ for any n and defines a norm.
Why is the definition above equivalent to taking the supremum only over functions with Lipschitz norm $\|f\|_L = 1$?

The solutions to these exercises will be discussed on Friday, 03.02.2017.