Workshop 'Perspectives in Optimal Transportation'

30 Nov – 02 Dec 2011

Ludwig-Maximilians Universität München

Speakers

M. Beiglböck (Vienna)	**C. Cotar (Toronto)**	**G. Friesecke (Munich)**	**P. Fuchs (Vienna)**
N. Gigli (Nice)	**N. Gozlan (Marne-La Vallee)**	**A. Guillin (Clermont-Ferrand)**	**M. Huessmann (Bonn)**
N. Juillet (Strassbourg)	**Ch. Leonard (Paris)**	**J. Maas (Bonn)**	**R. McCann (Toronto)**
R. Philipowski (Luxembourg)		**L. Rüschendorf (Freiburg)**	

Organizer: M. v. Renesse (LMU)

Conference Venue: Mathematisches Institut, Theresienstrasse 39, Room B 349, D-80333 München

Programme

Wed 30 Nov	Thurs 1 Dec	Fri 02 Dec
	09.00 – 09.50 Gigli **'On the interplay between horizontal and vertical derivation'**	09.00 – 09.50 Friesecke **'Density functional theory and optimal transportation with Coulomb cost (I)'**
	09.50 – 10.40 Juillet **'Diffusion by optimal transport in the Heisenberg group'**	09.50 – 10.20 Cotar **'Density functional theory and optimal transportation with Coulomb cost (II)'**
		10.20 – 10.50 Fuchs **'Quantum Fluid Models on the Space of Probability Measures'**
	10.40 – 11.10 (Coffee Break)	10.50 – 11.10 (Coffee Break)
	11.10 – 12.00 Maas **'Gradient flows of the entropy for finite Markov chains'**	11.10 –12.00 Leonard **'Entropic interpolations'**
	12.00 – 13.00 **(**Lunch)	12.00 – 12.50 McCann **'Hoelder continuity and injectivity of optimal maps'**
	13.00 – 13.50 Guillin **'Convergence to equilibrium for Fokker-Planck and granular media equations'**	12.50 – 13.00 (Closing)
14.00 – 14.30 (Registration)	13.50 – 14.40 Gozlan **'On Talagrand's and Log-Sobolev inequalities'**	
14.30 – 15.20 Rüschendorf **`Risk bounds and worst case dependence structure`**	14.40 – 15.30 (Transfer to TUM)	
15.20 – 16.10 Beiglböck **'Martingale Mass Transport and Robust Option Pricing'**	15.45 – 16.45 Dolzman* **'Analytical aspects of models in crystal plasticity'**	
16.10 – 17.00 (Coffee Break)	16.45 – 17.15 (Coffee Break)	
17.00 – 17.50 Huesmann **'Optimal transport between random measures'**	17.15 – 18.15 McCann* **'Geometric variational problems in economics: Optimal multidimensional pricing facing informational asymmetry'**	
17.50 – 18.40 Philipowski **'Ricci flow, coupling of Brownian motions and Perelman's L-functional'**	19.30 (Conference Dinner)	* Location: Boltzmannstr. 3 Room 03.06.011, 85748 Garching

Talk Titles and Abstracts

Mathias Beiglböck (Univ. Vienna)

Martingale Mass Transport and Robust Option Pricing

Abstract: Robust pricing of an exotic option Φ written on a financial asset can be viewed as the task of estimating $\E_\Q \Phi$, where $\Q$ runs through a set of martingale measures satisfying marginal constraints. It is fruitful to relate this to the theory mass transportation. E.g. the abstract duality theorem from optimal transport translates almost directly to new robust superheging results for stock price processes in discrete time. Joint work with F. Penkner and H. Labordaire, resp. N. Julliet.

Codina Cotar (Fields Inst. Toronto)

'Density functional theory and optimal transportation with Coulomb cost (II)'

Abstract: In this talk I explain a promising and previously unnoticed link between electronic structure of molecules and optimal tansportation, and first results. The `exact' mathematical model for electronic structure, the many-electron Schroedinger equation, becomes computationally unfeasible for more than a dozen or so electrons. For larger systems, the standard model underlying a huge literature in computational physics/chemistry/materials science is density functional theory (DFT). In DFT, one only computes the single-particle density instead of the full many-particle wave function. In order to obtain a closed equation, one needs a closure assumption which expresses the pair density in terms of the single-particle density ρ. We show that in the semiclassical limit, there holds an exact closure relation, namely the pair density is the solution to an optimal transport problem with Coulomb cost. We prove that this problem has a unique solution given by an optimal map; moreover we derive an explicit formula for the optimal map in the case when ρ is radially symmetric (note: atomic ground state densities are radially symmetric for many atoms such as He, Li, N, Ne, Na, Mg, Cu).
In my talk I focus on how to deal with its main mathematical novelties (cost decreases with distance; cost has a singularity on the diagonal). For the derivation of the Coulombic OT problem from the many-electron Schroedinger equation, see the talk by Gero Friesecke. Joint work with Gero Friesecke (TU Munich) and Claudia Klueppelberg (TU Munich)

Gero Friesecke (Techn. Univ. Munich)

'Density functional theory and optimal transportation with Coulomb cost (I)'

In this talk I explain a promising and previously unnoticed link between electronic structure of molecules and optimal tansportation, and first results. The `exact' mathematical model for electronic structure, the many-electron Schroedinger equation, becomes computationally unfeasible for more than a dozen or so electrons. For larger systems, the standard model underlying a huge literature in computational physics/chemistry/materials science is density functional theory (DFT). In DFT, one only computes the single-particle density instead of the full many-particle wave function. In order to obtain a closed equation, one needs a closure assumption which expresses the pair density in terms of the single-particle density ρ. We show that in the semiclassical limit, there holds an exact closure relation, namely the pair density is the solution to an optimal transport problem with Coulomb cost. We prove that this problem has a unique solution given by an optimal map;

moreover we derive an explicit formula for the optimal map in the case when ρ is radially symmetric (note: atomic ground state densities are radially symmetric for many atoms such as He, Li, N, Ne, Na, Mg, Cu). In my talk I focus on the derivation of the Coulombic OT problem from the many-electron Schroedinger equation. For how to deal with its main mathematical novelties (cost decreases with distance; cost has a singularity on the diagonal) see the talk by Codina Cotar. Joint work with Codina Cotar (Toronto) and Claudia Klueppelberg (TU Munich)

Philipp Fuchs (Techn. Univ. Vienna)

'Quantum Fluid Models on the Space of Probability Measures'

Abstract: Inspired by ideas of Felix Otto it was shown that a whole class of PDE can be expressed as gradient flows on the space of measures w.r.t. The Wasserstein metric. Moreover, as shown by Lott this space can, at least formally, be seen as a Riemannian manifold. This motivated the question what kind of mechanics could be done on P(M). Clearly, as we can only observe the evolution of densities (measures resp.), a full (fluid) mechanical model would not be reflected in this approach. However, there are so called quantum fluid models (where one is more interested in the evolution of the density) and we will show, that the PDE describing these models can be deduced to be the Euler-Lagrange-equations for some Lagrangian on Wasserstein space. This further leads to a nice connection to the Noether theorems

Nicola Gigli (Univ. Nice)

'On the interplay between horizontal and vertical derivation'

Abstract. I will talk about some recent developments on the interplay between the L^2 and W_2 geometries. Results include the possibility of studying the continuity equation in a genuine metric framework, and the identification of the heat flow as gradient flow in spaces with Ricci curvature bounded from below.

Nathael Gozlan (Univ. Marne-La Vallee)

'On Talagrand's and Log-Sobolev inequalities.'

Abstract: This talk is devoted to recent improvements of Otto-Villani theorem. The main result we will present states that Talagrand's inequality on a metric space (X,d) is equivalent to the Log-Sobolev inequality restricted to the class of log-d^2-convex functions. (Joint work with C. Roberto and P-M Samson)

Arnad Guillin (Univ. Clermont-Ferrand)

'Convergence to equilibrium for Fokker-Planck and granular media equations'

Abstract: We consider uniform convergence to equilibrium in Wasserstein distance for Fokker-Planck equations and nonlinear granular media equations. In particular, our results cover the long standing problem of non convex potentials. (with F. Bolley (Dauphine), I. Gentil (ICJ))

Martin Huesmann (Univ. Bonn)

'Optimal transport between random measures'

Abstract: We study couplings $q^\bullet$ between two jointly invariant random measures $\lambda^\bullet$ and $\mu^\bullet$ on $\R^d$, i.e. measure valued random variables $\omega\mapsto q^\omega$ and q^ω is a coupling between λ^ω and μ^ω. We look for minimizers of the mean transportation cost $$\mathfrak C(q^\bullet) = \sup_{B\subset \R^d} \frac1{Leb(B)} \mathbb{E}\int_{\R^d\times B} c(x,y)q^\omega(dx,dy).$$ If the mean transportation cost is finite and $\lambda^\omega \ll Leb$ then we prove the existence of a unique invariant coupling minimizing the mean transportation cost. If $\lambda\equiv Leb$, μ is a Poisson process and $c(x,y)=|x-y|^2$ the optimal coupling induces a random tiling of $\R^d$ into convex polytopes of volume 1.

Nicolas Juillet (Univ. Strassbourg)

'Diffusion by optimal transport in the Heisenberg group'

Abstract: In this talk, we will present the subRiemannian Heisenberg group H and its hypoelliptic diffusion, the ``heat diffusion" of H. We will show that in the Wasserstein space $P_2(H)$, it is a curve driven by the gradient flow of the Boltzmann entropy Ent. Conversely any gradient flow curve of Ent satisfy the hypoelliptic heat equation (without any additional assumption).

Christian Leonard (Univ. Paris Ouest)

'Entropic interpolations'

Abstract: McCann's displacement interpolations between probability measures solve a quadratic optimal transport problem. Similarly, we can consider interpolations which minimize some entropy functional. This statistical physics problem was addressed by Schrödinger in 1931, motivated by an analogy with quantum mechanics. We compare some aspects of these parallel notions of interpolation. For instance, both lead to the same Sturm-von Renesse criterion for lower bounded Ricci curvature on a Riemannian manifold.

Jan Maas (Univ. Bonn)

'Gradient flows of the entropy for finite Markov chains'

Absract: Since the work of Jordan, Kinderlehrer, and Otto in the 1990s it is known that the heat flow in R^n is the gradient flow of the entropy in the Wasserstein space of probability measures. In this talk, I will present a discrete counterpart to this result: given a reversible Markov kernel on a finite set, there exists a Riemannian metric on the space of probability measures, such that the corresponding Markov chain evolves as the gradient flow of the entropy. This can be used to define a notion of Ricci curvature of discrete spaces in the spirit of Lott, Sturm, and Villani. The Ricci curvature of the n-dimensional discrete hypercube turns out to be 2/n. This is joint work with Matthias Erbar (Bonn)

Robert McCann (Univ. Toronto)

'Hoelder continuity and injectivity of optimal maps'

Abstract: The Monge-Kantorovich optimal transportation problem is to pair producers with consumers so as to minimize a given transportation cost. Following work of Caffarelli and others on the quadratic cost, Ma-Trudinger and Wang found conditions on a general cost which guarantee the optimal map to be a diffeomorphism, assuming both mass distributions are smooth. Instead I will describe a conclusion which persists under a weaker hypotheses: namely that the optimal map remains a C^α_{loc} homeomorphism if the mass distributions are merely bounded above and below. The Hoelder exponent α turns out to be uniform among (A3w) costs, depending only on the bounds for the distributions. Joint work with Alessio Figalli and Young-Heon Kim.

Robert McCann (Univ. Toronto)

'Geometric variational problems in economics: Optimal multidimensional pricing facing informational asymmetry'

Abstract: The monopolist's problem of deciding what types of products to manufacture and how much to charge for each of them, knowing only statistical information about the preferences of an anonymous field of potential buyers, is one of the basic problems analyzed in economic theory. The solution to this problem when the space of products and of buyers can each be parameterized by a single variable (say quality X, and income Y) garnered Mirrlees (1971) and Spence (1974) their Nobel prizes in 1996 and 2001. The multidimensional version of this question is a largely open problem in geometric analysis (see Basov's book "Multidimensional Screening".) I plan to describe recent work with A Figalli and Y-H Kim, identifying structural conditions on the value b(X,Y) of product X to buyer Y which reduce this problem to a convex program in a Banach space--- leading to uniqueness and stability results for its solution, confirming robustness of c ertain economic phenomena observed by Armstrong (1996) such as the desirability for the monopolist to raise prices enough to drive a positive fraction of buyers out of the market, and yielding conjectures about the robustness of other phenomena observed Rochet and Chone (1998), such as the clumping together of products marketed into subsets of various dimension. The passage to several dimensions relies on ideas from differential geometry / general relativity, optimal transportation, and nonlinear PDE.

Robert Philipowski (Univ. Luxembourg)

'Ricci flow, coupling of Brownian motions and Perelman's L-functional'

Abstract: In this talk I will show that on a manifold whose Riemannian metric evolves under backwards Ricci flow two Brownian motions can be coupled in such a way that their normalized L-distance is a supermartingale. As a corollary, one obtains a new proof and a generalization of a recent result of Peter Topping concerning L-optimal transport.This is joint work with Kazumasa Kuwada.

Ludger Rüschendorf (Univ. Freiburg)

`Risk bounds and worst case dependence structure`