

### 13. Exercise sheet Algebraic Geometry I

All solutions have to be completely justified.

**Aufgabe 1** Let  $X$  and  $Y$  be two schemes over a scheme  $S$ . Let  $\{X_i\}_{i \in I}$  be an open covering of  $X$  and assume that  $X_i \times_S Y$  exists for all  $i \in I$ . Prove that  $X \times_S Y$  exists.

**Aufgabe 2** Let  $X$  and  $Y$  be two schemes over a scheme  $S$ .

(a) Show that if  $X \rightarrow S$  is a closed immersion, then the projection  $X \times_S Y \rightarrow Y$  is a closed immersion.

(b) Show that if  $X \rightarrow S$  is separated, then the projection  $X \times_S Y \rightarrow Y$  is separated.

**Aufgabe 3** Let  $X$  be a separated scheme over an affine scheme  $S$ . Let  $U$  and  $V$  be two open affine subschemes of  $X$ . Show that  $U \cap V$  is affine. Give an example to show that this fails if  $X$  is not separated.