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1. Exercise sheet Algebraic Geometry I

Aufgabe 1 Complete the proof of

Proposition: Let X be a non-empty topological space. The following are equivalent:

(i) X is irreducible.

(ii) Any two non-empty open subsets of X have a non-empty intersection.

(iii) Every non-empty open subset is dense in X.

(iv) Every non-empty open subset is connected.

(v) Every non-empty open subset is irreducible.

In the lecture we have shown (i) $\iff$ (ii) $\iff$ (iii).

Aufgabe 2 (a) Let $Y = V(T_2 - T_1^2)$ and set $\Gamma(Y) := k[T_1, T_2]/I(Y)$. Show that $\Gamma(Y)$ is isomorphic to a polynomial ring in one variable.
(b) Let $Z = V(T_1T_2 - 1)$. Show that $\Gamma(Z)$ is not isomorphic to a polynomial ring in one variable.

Aufgabe 3 (a) Compute the closure of $\mathbb{N} \subseteq \mathbb{A}^1(\mathbb{C})$.
(b) Show that for $X \subseteq \mathbb{A}^n(k)$ irreducible the product $X \times \mathbb{A}^1(k)$ is irreducible.

Aufgabe 4 Find the vanishing ideal $I(S)$ of $S = \{(1,2),(3,4)\} \subseteq \mathbb{A}^2(k)$.