Mathematisches Institut
der Universität München
Prof. Dr. B. Pareigis

SS 2002
Set 7

Problem set for
Quantum Groups and Noncommutative Geometry

(25) Let the set Z together with the multiplication $m : Z \times Z \longrightarrow Z$ be a monoid. Show that the unit element $e \in Z$ is uniquely determined.
Let (Z, m) be a group. Show that also the inverse $i : Z \longrightarrow Z$ is uniquely determined.
Show that unit element and inverses of groups are preserved by maps that are compatible with the multiplication.

(26) Find an example of monoids Y and Z and a map $f : Y \longrightarrow Z$ with $f(y_1 y_2) = f(y_1)f(y_2)$ for all $y_1, y_2 \in Y$, but $f(e_Y) \neq e_Z$.

(27) Let (G, m) be a group in $\mathcal{C}$ and $i_X : G(X) \longrightarrow G(X)$ be the inverse for all $X \in \mathcal{C}$. Show that i is a natural transformation.
Show that the Yoneda Lemma provides a morphism $S : G \longrightarrow G$ such that $i_X = \mathrm{Mor}_{\mathcal{C}}(X, S) = S(X)$ for all $X \in \mathcal{C}$.
Formulate and prove properties of S of the type $S * \mathrm{id} = \ldots$.

(28) Let $\mathcal{C}$ be a category with finite products. Show that a morphism $f : G \longrightarrow G'$ in $\mathcal{C}$ is a homomorphism of groups if and only if

$$\begin{tikzcd} G \times G \arrow[r, "m"] \arrow[d, "f \times f"'] & G \arrow[d, "f"] \\ G' \times G' \arrow[r, "m'"] & G' \end{tikzcd}$$

commutes.

Due date: Tuesday, 11.06.2002, 16:15 in Lecture Hall E41