

Problem set for  
Advanced Algebra

(37) Let  $f, g : X \rightarrow Y$  be two maps. Show that the set  $\{x \in X \mid f(x) = g(x)\}$  with the inclusion map into  $X$  is an equalizer of  $f, g : X \rightarrow Y$ .

(38) (a) Let the commutative diagram

$$\begin{array}{ccc} P & \xrightarrow{p} & A \\ q \downarrow & & \downarrow f \\ B & \xrightarrow{g} & C \end{array}$$

be a pullback (a limit) of the morphisms  $f : A \rightarrow C$  and  $g : B \rightarrow C$ . Assume that  $g$  is a monomorphism. Show that  $p$  is also a monomorphism.

(b) Show that the category of sets has pullbacks.

(39) Let  $X$  and  $Y$  be two sets. Show that the disjoint union  $X \dot{\cup} Y$  is a coproduct of  $X$  and  $Y$  in the category of sets.

(40) Show that the category of sets has coequalizers.