LMU Munich
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Exercises on Mathematical Statistical Physics
Physics Sheet 4

Problem 1 (Two Particle Problem). We consider two electrons in two dimensions in the presence of a perpendicular uniform magnetic field interacting via a potential $V(\vec{r}_1 - \vec{r}_2)$. The two-body Hamiltonian is:

$$H = \hbar\omega_c(a_1^\dagger a_1 + a_2^\dagger a_2) + V(\vec{r}_1 - \vec{r}_2),$$

where a_i are the individual ladder operators we defined in class and $\vec{r}_i$ are the particle coordinates. Let us define ladder operators $a_R(a_R^\dagger)$ and $a_r(a_r^\dagger)$ for the center of mass and the relative particle, respectively, in the form

$$a_R = \frac{a_1 + a_2}{\sqrt{2}} \tag{1}$$

$$a_r = \frac{a_1 - a_2}{\sqrt{2}}. \tag{2}$$

(a) Show that the center of mass operators commute with the relative particle ones. Show that $H = \hbar\omega_c(a_R^\dagger a_R + a_r^\dagger a_r) + V(\vec{r}_1 - \vec{r}_2)$.

(b) Let us define center orbit ladder operators b_R, b_r for the center of mass and the relative coordinate, related to the individual ladder operators b_i in the same way as for a_R, a_r above. Consider the symmetric gauge. Show that b_r only involves the relative coordinate $z = z_1 - z_2$ and that its spatial form is identical to the one of the one-particle operator b_i with the replacements $z_i \to z$ and $\ell \to \ell_r = \sqrt{2}\ell$. Similarly, show that b_R only involves the center of mass coordinate $Z = (z_1 + z_2)/2$, and that the effective magnetic length is $\ell_R = \ell/\sqrt{2}$.

(c) Write down the expression for the two-body eigenstates of Hamiltonian H in the lowest Landau level. What are the corresponding eigenvalues E_k? If the number of fluxes piercing the sample is N_ϕ, what is the degeneracy g_k of each eigenvalue? Check that the total number of two-particle states within the lowest Landau level coincides with $\sum_k g_k$.

(d) Express the exact lowest Landau level two-body eigenstate

$$\psi(z_1, z_2) = (z_1 - z_2)^3 e^{-\frac{|z_1|^2 + |z_2|^2}{4}}$$

in terms of the basis of all possible two-body Slater determinants.

Problem2 (Laughlin states). Let us consider the Laughlin state at filling factor $\nu = 1/m$:

$$\Psi = \alpha \prod_{i<j}^{N} (z_i - z_j)^m e^{-\sum_i^N |z_i|^2/4},$$

where α is the corresponding normalization factor, $z = (x + iy)/\ell$ and N is the number of particles.

(a) Estimate the number of different Slater determinants that participate in the state Ψ. How does this number behave as the number of particles increases?

(b) Consider the problem of encoding the many-body state Ψ in a classical computer. How do memory resources needed increase with the number of particles?

(c) Consider the problem of calculating the normalization factor α. Estimate the complexity of this calculation as a function of the number of particles.

Problem 3 (Bosonic Laughlin states). Let us consider the bosonic Laughlin state

$$\Psi = \alpha \prod_{i<j}^{N} (z_i - z_j)^2 e^{-\sum_i^N |z_i|^2/2},$$

with $z = (x + iy)/\ell$. Show that the state Ψ is an exact eigenstate of the many-body Hamiltonian:

$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2 - \Omega \vec{r}_i \times \vec{p}_i + g \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j),$$

with $\ell^2 = \hbar/m\omega$. What is the corresponding eigenvalue?

The solutions to these exercises will be discussed on Friday, 15th July, and Monday, 18th July.