ON CONSTRAINED RANDOM GRAPHS

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In this talk we consider random graphs subject to topological constraints. We compare two *probability distributions* on the set of simple graphs on n vertices induced by a given constraint:

(1) The *microcanonical ensemble*, where the constraint is *hard*, i.e., has to be satisfied for every realisation of the graph.

(2) The *canonical ensemble*, where the constraint is *soft*, i.e., has to be satisfied on average.

Two regimes are of interest:

(a) The *sparse regime*, where the number of edges per vertex remains bounded as $n \to \infty$.

(b) The *dense regime*, where the number of edges per vertex diverges as $n \to \infty$.

We say that *breaking of ensemble equivalence* occurs in the limit as $n \to \infty$ when the *relative entropy* of the two ensembles per vertex (in the sparse regime), respectively, per edge (in the dense regime) is strictly positive. We present various examples of constraints where breaking of ensemble equivalence occurs, e.g. when the constraint is on the degree sequence or when the constraint is on the total number of edges and triangles.

Breaking of ensemble equivalence is important, because it says that if we use random graphs to model a real-world network based on *partial information* about the architecture of the network, then we need to be very careful what model to choose.

Joint work with Diego Garlaschelli (Leiden), Michel Mandjes (Amsterdam), Andrea Roccaverde (Leiden), Tiziano Squartini (Rome), Nicos Starreveld (Amsterdam).

The talk is aimed at a general mathematics audience and does not presume any prior knowledge of random graph theory.