Colloquium and Workshop Honouring Wilfried Buchholz

April 4–5, 2008, München

Time	Friday (Room B051)	Saturday (Room B138)	Sunday (Room B349)
9.00			
9.15			
9.30		**Coffee**	**Breakfast** Following a tradition, a Bavarian Weißwurstfrühstück will be serverd in the institute for those who wish. Other breakfast will be available as well. The breakfast will finish well in time to get trains at noon.
9.45			
10.00		**Opening**	
10.15		*Herman Ruge Jervell:* Finite trees as ordinals	
10.30			
10.45		*Andreas Abel:* Normalization by Evaluation for Martin-Löf Type Theory	
11.00			
11.15		*Anton Setzer:* The strength of Martin-Löf Type Theory with the logical framework	
11.30			
11.45		**Coffee**	
12.00			
12.15		*Michael Rathjen:* Π^0_2 Conservation in Proof Theory	
12.30			
12.45			
13.00		**Lunch Break**	
13.15			
13.30			
13.45			
14.00			
14.15		**Coffee**	
14.30			
14.45			
15.00		*Ryota Akiyoshi:* An ordinal-free proof of cut-elimination theorem for Π^1_1-analysis with ω-rule	
15.15			
15.30		*Dieter Probst:* Π_3-reflection in Kripke-Platek set theory and nonmonotone inductive definitions from the class $[\Pi^0_1, \dots, \Pi^0_1]$.	
15.45	**Coffee**		
16.00		**Coffee**	
16.15	*Stan Wainer:* Tree Ordinals and Proof Theoretic Bounding Functions		
16.30		*Daria Spescha:* Elementary explicit types and polynomial time operations	
16.45			
17.00	**Coffee**	*Luca Alberucci:* The modal μ-Calculus Hierarchy over restricted Classes of Transition Systems	
17.15			
17.30	*Wolfram Pohlers:* Ordinal Notations and Controlling Operators	**Coffee**	
17.45			
18.00		*Gerhard Jäger:* Towards a Proper Proof Theory of the Modal μ-Calculus	
18.15	**Reception**		
18.30			
18.45	*Franz-Viktor Kuhlmann:* What is the connection between resolution of singularities and decidability?	**Coffee**	
19.00			
19.15	*Matthias Baaz:* Cut-elimination by resolution and interpolation	*Arnold Beckmann:* Proof Notations for Bounded Arithmetic	
19.30			
19.45	Informal discussions with **more coffee**; the welcome dinner is next door to the institute.	**Coffee**	
20.00			
20.15			
20.30	**Welcome Dinner** à la carte at "Taverna Olympos"	**Conference Dinner** à la carte at "Weißes Bräuhaus"	
20.45			
21.00			
21.15			
21.30			
21.45			
22.00			

Program Day by Day

Friday, April 4, Room B051

15:45 - 16:15	**Coffee**
16:15 - 17:00	*Stan Wainer:* Tree Ordinals and Proof Theoretic Bounding Functions
17:00 - 17:30	**Coffee**
17:30 - 18:15	*Wolfram Pohlers:* Ordinal Notations and Controlling Operators
18:15 - 18:45	**Reception**
18:45 - 19:15	*Franz-Viktor Kuhlmann:* What is the connection between resolution of singularities and decidability?
19:15 - 19:45	*Matthias Baaz:* Cut-elimination by resolution and interpolation
19:45 - 20:30	Informal discussions with **more coffee**; the welcome dinner is next door to the institute.
20:30 - 22:00	**Welcome Dinner** à la carte at "Taverna Olympos"

Saturday, April 5, Room B138

09:30 - 10:00	**Coffee**
10:00 - 10:15	**Opening**
10:15 - 10:45	*Herman Ruge Jervell:* Finite trees as ordinals
10:45 - 11:15	*Andreas Abel:* Normalization by Evaluation for Martin-Löf Type Theory
11:15 - 11:45	*Anton Setzer:* The strength of Martin-Löf Type Theory with the logical framework
11:45 - 12:15	**Coffee**
12:15 - 13:00	*Michael Rathjen:* Π^0_2 Conservation in Proof Theory
13:00 - 14:15	**Lunch Break**
14:15 - 15:00	**Coffee**

15:00 - 15:30	*Ryota Akiyoshi:* An ordinal-free proof of cut-elimination theorem for Π^1_1-analysis with ω-rule
15:30 - 16:00	*Dieter Probst:* Π_3-reflection in Kripke-Platek set theory and nonmonotone inductive definitions from the class $[\Pi^0_1, \ldots, \Pi^0_1]$.
16:00 - 16:30	**Coffee**
16:30 - 17:00	*Daria Spescha:* Elementary explicit types and polynomial time operations
17:00 - 17:30	*Luca Alberucci:* The modal μ-Calculus Hierarchy over restricted Classes of Transition Systems
17:30 - 18:00	**Coffee**
18:00 - 18:45	*Gerhard Jäger:* Towards a Proper Proof Theory of the Modal μ-Calculus
18:45 - 19:15	**Coffee**
19:15 - 19:45	*Arnold Beckmann:* Proof Notations for Bounded Arithmetic
19:45 - 20:15	**Coffee**
20:30 - 24:00	**Conference Dinner** à la carte at "Weißes Bräuhaus"

Sunday, April 6, Room B349

09:30 - 11:15	**Breakfast** Following a tradition, a Bavarian Weißwurstfrühstück will be serverd in the institute for those who wish. Other breakfast will be available as well. The breakfast will finish well in time to get trains at noon.

Abstracts

Andreas Abel: *Normalization by Evaluation for Martin-Löf Type Theory*

The decidability of equality is proved for Martin-Löf type theory with a universe à la Russell and typed $\beta\eta$-equality judgements. A corollary of this result is that the constructor for dependent function types is injective, a property which is crucial for establishing the correctness of the type-checking algorithm. The decision procedure uses normalization by evaluation, an algorithm which first interprets terms in a domain with untyped semantic elements and then extracts normal forms. The correctness of this algorithm is established using a PER-model and a logical relation between syntax and semantics.

Ryota Akiyoshi: *An ordinal-free proof of cut-elimination theorem for* Π^1_1*-analysis with* ω*-rule*

Tait proposed a quite simple ordinal-free cut-elimination proof for Π^1_1 analysis by analysing of Takeuti's one. The aim of our talk is to report our progress on a relationship between Tait's cut-elimination proof and Buchholz' one, more precisely, to explain Tait's cut-elimination proof in terms of Buchholz' Ω-rule.

(joint work with G. Mints, work in progress)

Luca Alberucci: *The modal* μ*-Calculus Hierarchy over restricted Classes of Transition Systems*

Abstract: We analize the modal μ-Calculus Hierarchy over various classes of Transition Systems and idenitify some classes where the hierarchy collapses or remains strict.

Matthias Baaz: *Cut-elimination by resolution and interpolation*

Using methods from cut-elimination by resolution we show, that for any LK-derivation with cuts of $A \Rightarrow B$ there is a number of pre-interpolants $I_1, \ldots, I_n$ (n linear in the length of the proof), such that there is an interpolant of $A \Rightarrow B$, which is a Boolean combination of substitution instances of $I_1, \ldots, I_n$.

Arnold Beckmann: *Proof Notations for Bounded Arithmetic*

We inroduce a Buchholz-style notation system [Buc91] for the class of propositional proofs which are obtained by translating formal proofs in Bounded Arithmetic to propositional logic. The propositional translation used here is well-known in proof-theoretic investigations. In the Bounded Arithmetic community this translation is known as the *Paris-Wilkie-translation*. Employing the fact that cut-reduction operates feasibly on proof notations [AB07], we will explain how this setting can be used to obtain new uniform proofs of various known characterisations of definable functions in Bounded Arithmetic.

[AB07] Klaus Aehlig and Arnold Beckmann. On the computational complexity of cut-reduction. Accepted for publication at LICS 2008. Full version available as Technical Report CSR15-2007, Department of Computer Science, Swansea University, December 2007. `http://arxiv.org/abs/0712.1499`.

[Buc91] Wilfried Buchholz. Notation systems for infinitary derivations. *Archive for Mathematical Logic*, 30:277–296, 1991.

Herman Ruge Jervell: *Finite trees as ordinals*

We give a simple wellordering of all finite trees. The wellordering corresponds to the n-ary Veblen hierarchy where we enumerate without fix points.

Franz-Viktor Kuhlmann: *What is the connection between resolution of singularities and decidability?*

Completeness and decidability of mathematical theories are a main subject in model theory. They have been of particular interest in model theoretic algebra. There are nice examples from field theory that by now may well be called "classical". Model theoretical results for algebraically closed, real closed and p-adically closed fields have found interesting applications: Hilbert's 17. Problem, description of positive definite polynomials and their p-adic analogues, Nullstellensätze. In the year 1965 Ax and Kochen generated much interest in model theory through their proof of a correct version of Artin's Conjecture about non-trivial zeros of forms over the p-adic numbers. Since then one of the best known open problems in model theoretic algebra is whether the elementary theory of the field $\mathbb{F}_p((t))$ of formal Laurent series over the field with p elements is decidable. Although this field looks so similar to the field $\mathbb{Q}_p$ of p-adic numbers and the theory of the latter has been shown by Ax, Kochen and Ershov to be decidable, several excellent model theorists tried in vein to solve this problem. But this is not due to a lack of knowledge in model theory. In contrast to $\mathbb{Q}_p$, the field $\mathbb{F}_p((t))$ is a valued field of positive characteristic, and we simply do not know enough about the structure of such valued fields. In this way, model theoretical questions have stimulated new research in a classical area of algebra: valuation theory.

Again in 1965, another famous theorem was proved: Hironaka showed resolution of singularities for all algebraic varieties over fields of characteristic 0. Since then also for this theorem its analogue in positive characteristic has remained an open problem, in spite of all attacks from excellent algebraic geometers. Since Zariski it is known that the local version of resolution of singularities, called "local uniformisation", is of valuation theoretical nature. Yet it was a quite unexpected finding that the decidability problem and the problem of local uniformisation both are based on the same valuation theretical problem: the defect. In the presence of defect, the classification of valued fields up to elementary equivalence, relative to their invariants (value group and residue field), breaks down.

Another good indication for the connection between the two problems is the work of Denef and Schoutens. They show that if resolution of singularities in positive characteristic holds, then at least the existential elementary theory of $\mathbb{F}_p((t))$ is decidable.

Dieter Probst: *Π_3-reflection in Kripke-Platek set theory and nonmonotone inductive definitions from the class $[\Pi^0_1, \ldots, \Pi^0_1]$.*

When speaking of reflection, it is important to specify on what one reflects. The simplest form of Π_2-reflection in Kripke-Platek set theory is captured by the principle (R_1) that claims the existence of a reflecting set which is transitive, i.e. for each Π_2 formula $A(u)$,

$$(R_1) \qquad A(x) \to \exists a[\mathsf{trans}(a) \land x \in a \land A^a(x)].$$

n-times iterated Π_2-reflection is then the principle (R_{n+1}) asking for the existence of a reflecting set that satisfies also all instances of (R_n). $\mathsf{KPu}^0 + (R_2)$, for instance, is basically KPm^0.

In this talk, we argue how to obtain a proof-theoretic analysis of Π_3-reflection on transitive sets (or admissibles): This theory straightforwardly reduces to iterated Π_2-reflection which in turn reduces to theories for non-monotone inductive definitions of the form $[\Pi^0_1, \ldots, \Pi^0_1]$. Such an operator form is specified by Π^0_1 operator forms $A_i(\mathsf{P}, x)$ $(0 \leq i < n)$ and its extension $I := \bigcup_{\gamma \in \mathsf{ON}} I^\gamma$ is given by setting $I^\gamma := I^{<\gamma} \cup F^{A_i}(I^{<\gamma})$, where $I^{<\gamma}$ is already closed under all the operators F^{A_j} $(j < i)$. We sketch how to embed theories for such non-monotone inductive definitions with restricted fixed-point induction into subsystems of second order arithmetic featuring iterated Π^1_2-reflection, for which we already have an ordinal analysis at hand.

Michael Rathjen: Π^0_2 *Conservation in Proof Theory*

Proof theory of infinitary derivations gives rise to finitistic reductions, especially Π^0_2 conservation results. How is this achieved? The conservative extension statement itself is a Π^0_2 sentence. Does it have a reasonable Skolem function? Wilfried Buchholz's work has been a beacon of clarity in answering these questions.

Anton Setzer: *The strength of Martin-Löf Type Theory with the logical framework*

The logical framework is a dependently typed lambda-calculus which is added on top of set. The formulation of Martin-Löf type theory is simplified in the presence of the logical framework. Although it is considered as folklore that the logical framework doesn't add any real strength, we have avoided it until now when carrying out proof theoretic analyses of variants of Martin-Löf type theory, because our approach to modelling Martin-Löf type theory in variants of Kripke-Platek set theory didn't allow to integrate the logical framework, and our understanding of this folklore result didn't suffice in order to guarantee that we indeed obtain the same proof theoretic strength.

In this talk we will show how to overcome those limitations, and to determine upper bounds for the strenght for Martin-Löf type theory with the W-type and one universe or one Mahlo universe.

Daria Spescha: *Elementary explicit types and polynomial time operations*

Our resarch addresses systems of explicit mathematics as first introduced by Feferman in 1975. We propose weak explicit type systems with a restricted form of elementary comprehension whose provably terminating operations coincide with the functions on binary words that are computable in polynomial time. The systems considered are natural extensions of the first-order applicative theories introduced by Strahm. We also present several extensions that are mostly based on a theory by Cantini. In particular, we study a natural application of Cantini's uniformity principle.

(joint work with Thomas Strahm)

List of Participants

Andreas Abel
Klaus Aehlig
Ryota Akiyoshi
Luca Alberucci
Matthias Baaz
Sebastian Bauer
Arnold Beckmann
Ulrich Berger
Peter Berry
Wilfried Buchholz
Marco Denini
Birgit Elbl
Simon Huber
Gerhard Jäger
Herman Ruge Jervell
Franz-Viktor Kuhlmann
Markus Latte
Hans Leiß
Grisha Mints
Karl-Heinz Niggl
Peter Päppinghaus
Wolfram Pohlers
Dieter Probst
Florian Ranzi
Michael Rathjen
Peter Schuster
Helmut Schwichtenberg
Monika Seisenberger
Anton Setzer
Daria Spescha
Nik Sultana
Stan Wainer
Albert Ziegler
Wolfgang Zuber

Excerpt from the Mathematical Genealogy