AB Geometrie und Topologie
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Topology II
SoSem 2026
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The course continues the "Topology I" from the previous semester where we discussed the fundamental group, covering spaces and the beginnings of singular homology. The focus will now lie on homology and cohomology.

Whereas the basic algebro-topological invariants of homotopic nature, the homotopy groups, are very naturally and directly defined, their elements being deformation classes of maps, computing them is often very hard. The most basic and important of these invariants, the fundamental group studied in the previous semester, is a non-abelian object. This makes it a strong invariant able to carry much information, the price being that in general it is hard to evaluate (even after being computed) because a description of a group such as a presentation by generators and relations as provided by the Seifert-van Kampen theorem does not give away its secrets easily.

The homological invariants on the other hand are conceptually more involved to define but much more accessible to computation, due to their essential feature that they keep track of unordered arrangements of "small pieces" and can thus be localized (for instance, Mayer-Vietoris is much more generally applicable and easier to work with than Seifert-van Kampen!) and also because they are algebraically easier to deal with being modules over commutative rings or, in the case of cohomology which comes with a product structure, graded-commutative algebras.

We plan to discuss the following main topics:

- singular homology and cohomology, along with some homological algebra

- CW-complexes and cellular (co)homology

- manifolds and duality

We will begin with making the connection to the discussion of singular homology at the end of previous semester and soon give some of the classical applications.

It is planned to continue the course in the winter term and as of now a good candidate for the topic there are vector bundles and characteristic classes.

For students of mathematics/physics, third year or later (bachelor, master, TMP).

Prerequisites: Topology I.

References: Hatcher, *Algebraic topology*, Cambridge University Press, 2002

Munkres, *Elements of algebraic topology*, Chapman & Hall, 1984

Bredon, *Topology and geometry*, Springer, 1993

Greenberg, Harper, *Algebraic topology: a first course*, Addison-Wesley, 1981