AB Geometrie und Topologie
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(Algebraic) Topology I

WiSem 2025/2026

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A guiding question in topology is to distinguish topological spaces up to homeomorphism. Since continuous maps are so flexible, it is usually by no means evident how to rule out the existence of a homeomorphism between two given spaces, for instance, between the sphere and the torus. The basic approach of algebraic topology is to try to "gain clarity" by passing from spaces and maps to "deformation classes" of such. This turns out to be stepping from a continuous to a discrete world, from geometry to algebra, and produces algebraic invariants (groups, rings, ...) which in principle are computable (and sometimes also in practice). One of the first places where one encounters this phenomenon is basic complex analysis; when defining winding numbers one passes from the entirety of all closed curves in the punctured plane to integers.

The two main topics of the course will be:

- fundamental group and covering spaces
- singular homology

We will begin with a bit of algebra and discuss first some basic notions from group theory which usually are not covered in the beginner's courses, namely free groups and presentations of groups by generators and relations.

The course will be continued in the summer term where the main topics will be singular homology and cohomology with coefficients along with the necessary results from homological algebra.

For students of mathematics/physics, third year or later (bachelor, master, TMP).

Prerequisites: basic courses in calculus (=Analysis) and linear algebra, basic knowledge of general topology (alias point set topology).

References:

A. Hatcher, *Algebraic topology*, Cambridge University Press, 2002

W.S. Massey, *Algebraic topology: An introduction*, GTM 56, Springer, 1967