

# Compact Lie groups and their representations

SEMINAR WiSEM 2025/2026

as of July 16, 2025

Lie groups are “smooth groups”, i.e. they are simultaneously groups and smooth manifolds. Both structures are compatible in the sense that the group operations are differentiable. Important examples are matrix groups like the general linear groups  $GL(n, \mathbb{R})$ , the special linear groups  $SL(n, \mathbb{R})$  and the orthogonal groups  $O(n)$ . Lie groups arise as continuous symmetries and they were discovered in the 19th century by the norwegian mathematician Sophus Lie when he investigated the symmetries of differential equations. They play now a basic role in all of mathematics and physics.

A representation of an abstract group is a realization as a group of linear automorphisms of a vector space. Representation theory studies the question in which ways a given group can operate linearly on vector spaces.

The seminar will focus on compact Lie groups and their representations. The aim will be the classification of the irreducible representations of a compact Lie group by their highest weights and their construction (Weyl character formula, Borel-Weil theorem). We will put emphasis on examples and explicitly describe the classical groups and their irreducible representations.

**Main reference:** Bröcker, tom Dieck, *Representation theory of compact Lie groups*, GTM 98, Springer, 1985

**Prerequisites:** Analysis I-III and Linear Algebra I-II, basic knowledge of differentiable manifolds

**Language:** English.

**For:** students of mathematics or physics (bachelor, master, TMP)

**Time+room:** Tuesday 4:15-6:00 pm in room B 252

**First talk:** 14.10.

**Registration:** Please write me e-mail as soon as possible if you want to participate, and in any case **before** the first talk.