

# Lie groups: Compact Lie groups and their representations

SEMINAR SoSEM 2021

as of March 3, 2021

Lie groups are “smooth groups”, i.e. they are simultaneously groups and smooth manifolds. Both structures are compatible in the sense that the group operations are differentiable. Important examples are matrix groups like the general linear groups  $GL(n, \mathbb{R})$ , the special linear groups  $SL(n, \mathbb{R})$  and the orthogonal groups  $O(n)$ . Lie groups arise as continuous symmetries and they were discovered in the 19th century by the norwegian mathematician Sophus Lie when he investigated the symmetries of differential equations. They play now a basic role in all of mathematics and physics.

A representation of an abstract group is a realization as a group of linear automorphisms of a vector space. Representation theory studies the question in which ways a given group can operate linearly on vector spaces.

The seminar will focus on compact Lie groups and their representations. The aim will be the classification of the irreducible representations of a compact Lie group by their highest weights and their construction (Weyl character formula, Borel-Weil theorem). We will put emphasis on examples and explicitly describe the classical groups and their irreducible representations.

In large parts of the seminar, we will follow the book by Bröcker and tom Dieck.

**References:** Bröcker, tom Dieck, *Representation theory of compact Lie groups*, GTM 98, Springer, 1985

Sepanski, *Compact Lie groups*, GTM 235, Springer, 2007

Knapp, *Representation Theory of Semisimple Groups: An Overview Based on Examples*, Princeton, 1986

Fulton, Harris, *Representation theory - A first course*, GTM 129, Springer, 1991

**Prerequisites:** Analysis I-III and Linear Algebra I-II, basic knowledge on differentiable manifolds and Lie groups

**For** students of mathematics and physics (Bachelor, Master, TMP)

**Language:** German and/or english, depending on the participants

**Time:** Tuesday 16-18 (tentatively, to be discussed in the preliminary meeting).

**Modus:** Online via zoom (most likely, at least in the beginning) or in room A 027 (also to be discussed).

**Preliminary meeting:** Wednesday **march 3** at **16:00** via zoom.

**Registration:** If you are interested in participating, please write me email.