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Lineare Algebra II – Tutoriumsblatt 13

Aufgabe 1.

Let K be a field, $a \in K$, and $A_a := \begin{pmatrix} a+2 & a+1 \\ a^2-a-2 & a^2-a-1 \end{pmatrix} \in \mathrm{M}_2(K)$.

1. Find the Jordan normal form for A_a.
2. Find a Jordan basis for A_a.
3. Find the additive Jordan Decomposition for A_a.
4. Find all invariant subspaces of A_a.

Aufgabe 2.

For $a \in \mathbb{R}$ and $A_a := \begin{pmatrix} 2 & a \\ a & 2 \end{pmatrix} \in \mathrm{M}_2(\mathbb{R})$.

1. Find a basis such that A_a is diagonal in this basis.
2. For which a does A_a define a scalar product on $\mathbb{R}^2$?

Aufgabe 3.

1. For $v \in \mathbb{R}^3 \setminus 0$, $\theta \in [0, 2\pi[$, let $\rho(v,\theta)$ denote the rotation of $\mathbb{R}^3$ through the angle θ about the axis $\langle v \rangle$. Prove that there exists a basis of $\mathbb{R}^3$ such that $\rho(v,\theta)$ has the matrix

$$R(\theta) := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

   in this basis.
2. Prove that each element of $\mathrm{SO}_3(\mathbb{R})$ equals $\rho(v,\theta)$ for some $v \in \mathbb{R}^3 \setminus 0$, $\theta \in [0, 2\pi[$.