LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Lineare Algebra II – Tutoriumsblatt 12

Aufgabe 1.
Consider a rotation $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \in \mathrm{SO}_2(\mathbb{R})$. Decompose it as a product of two reflections.

Aufgabe 2.

1. Prove that the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ is diagonalize and give a basis of $\mathbb{R}^3$ consisting of eigenvectors of A.
   Remark: Conclude that A admits a square root.
2. Denote $\langle u, v\rangle_A := u^t A v$ the corresponding symmetric bilinear form. Prove that this form is positive-definite and find a basis v_1, v_2, v_3 such that the Gram matrix of this form $G(v_1, v_2, v_3) = (\langle v_i, v_j\rangle_A)_{ij}$ is diagonal.

Aufgabe 3.
Let $G(v_1, \ldots, v_n)$ denote the Gram matrix of the vectors $v_1, \ldots, v_n \in \mathbb{R}^n$, and $\sigma \in \mathfrak{S}_n$. Prove that $\det G(v_{\sigma(1)}, \ldots, v_{\sigma(n)}) = \det G(v_1, \ldots, v_n)$.

Aufgabe 4.
Let $f\colon V \to V$ be an isometry.

1. For an f-invariant subspace $U \leq V$ prove that $U^\perp$ is also f-invariant.
2. Prove that there exists a subspace $U \leq V$ such that V can be decomposed as an *orthogonal* direct sum of U and the eigenspaces $V_1(f)$ and $V_{-1}(f)$.