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Lineare Algebra II – Tutoriumsblatt 9

Aufgabe 1.

Find the Jordan normal form and a Jordan basis for the following matrix over $\mathbb{C}$.

1. $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix};$

2. $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$

Aufgabe 2.

Let K be a field, V a 4-dimensional K -vector space, and $f: V \rightarrow V$ a nilpotent endomorphism.

1. List all possible JNF for f .
2. List all possible sequences of dimensions of generalized eigenspaces $\text{Ker}(f^i)$ for $i > 0$, and explain how they correspond to the possible JNF.

Aufgabe 3.

Find the Jordan normal form and a Jordan basis for the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 9 & -3 & -6 & -6 \\ -5 & 2 & 4 & 3 \\ 3 & -1 & -2 & -2 \end{pmatrix}$$

over $\mathbb{C}$.

Hint: Compute A^2 .

Aufgabe 4.

For $A \in M_6(\mathbb{C})$ with $\text{tr } A = 3$ and $\text{rk } A = 4$ assume that $A^2(A^2 - E) = 0$. Find the Jordan normal form for A .