LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Prof. Dr. Fabien Morel

Dr. Andrei Lavrenov, Oliver Hendrichs, Katharina Novikov

Sommersemester 2025

28. April 2025

Lineare Algebra II – Tutoriumsblatt 1

Aufgabe 1.

1. Divide $4x^3 + x^2$ by $x + 1 + i$ in $\mathbb{C}[x]$.
2. Find gcd(6787, 7194).

Aufgabe 2.

1. Let A_1, A_2 be commutative rings, and $A := A_1 \times A_2$. Denote $e_1 := (1, 0)$ and $e_2 := (0, 1) \in A$. Then any A-module M decomposes as a direct sum of its submodules $M = M_1 \oplus M_2$, where $M_i := e_i M = \{e_i \cdot m \mid m \in M\} \subseteq M$, for $i = 1, 2$.
2. Let $A_1 = \mathbb{Z}/2$, $A_2 = \mathbb{Z}/3$, $M = \mathbb{Z}/6$. Prove that M has an A-module structure for $A = A_1 \times A_2$ via $(a, b) \cdot m := (3a + 4b)m$. Find M_1 and M_2.

Aufgabe 3.

1. For $f(x) \in \mathbb{C}[x]$ and $n \in \mathbb{N} \setminus 0$ assume that the polynomial $f(x^n)$ is divisible by $x - 1$ in $\mathbb{C}[x]$. Prove that then in fact $f(x^n)$ is divisible by $x^n - 1$.
2. Prove that every polynomial $f(x) \in \mathbb{R}[x]$ of odd degree has a root.
3. Prove that $f(x) = x^{3m} + x^{3n+1} + x^{3p+2}$ is divisible by $x^2 + x + 1$ in $\mathbb{Q}(x)$.
4. For which m, n, $p \in \mathbb{N}$ the polynomial $f(x) = x^{3m} - x^{3n+1} + x^{3p+2}$ is divisible by $x^2 - x + 1$ in $\mathbb{Q}(x)$?

Aufgabe 4.

1. Let $x = 111\ldots11$ be a number consisting of m "ones", and $y = 111\ldots11$ be a number consisting of n "ones". Find $\gcd(x, y)$.
2. Find $\gcd(x^n - 1, x^m - 1)$ in $\mathbb{C}[x]$.