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## Lineare Algebra I – Tutoriumsblatt 08

### Aufgabe 1.

Find a basis of the subspace  $U \leq \mathbb{R}^4$  and the quotient space  $\mathbb{R}^4/U$  with

$$U = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + 2x_2 = 0, x_3 + 3x_4 = 0\}.$$

### Aufgabe 2.

Prove that a linear form  $f$  is equal to a linear combination of forms  $f_1, \dots, f_k$  if and only if  $\bigcap_{i=1}^k \text{Ker}(f_i) \subset \text{Ker}(f)$ .

### Aufgabe 3.

Let  $W = V_1 \oplus V_2$  and let  $U$  be a 1-dimensional subspace of  $W$  such that  $U \cap V_1 = 0 = U \cap V_2$ . Find the dimension of  $(V_1 + U) \cap (V_2 + U)$ .

### Aufgabe 4.

Let  $V$  denote a vector space of dimension  $n$  over a field  $K = \mathbb{Z}/p\mathbb{Z}$ .

1. Let  $u_1, \dots, u_k \in V$  be linearly independent vectors. How many elements has a subspace  $U$  generated by  $u_1, \dots, u_n$ ?
2. How many bases admit  $V$ ?