LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Lineare Algebra I – Tutoriumsblatt 04

Aufgabe 1.

1. Let $(G, \cdot)$ be a group. For $g, h \in G$ denote $g * h := h \cdot g$. Prove that $(G, *)$ is a group isomorphic to $(G, \cdot)$.
2. Let $G = (G, \cdot)$ be a group and assume that $(\ )^{-1} \colon G \to G$ is a group homomorphism. Prove that G is abelian.

Aufgabe 2.
Let U be a set. Prove that $(\mathcal{P}(U), \triangle)$ is a group. You can use Aufgabe 1 from Tutoriumsblatt 1.

Aufgabe 3.

1. Prove that the set of bijections $\mathfrak{S}_X$ of a set X form a group with respect to composition.
2. For a subset $Y \subset X$, any $g \in \mathfrak{S}_Y$ defines an element $i(g) \in \mathfrak{S}_X$ as follows:

$$i(g)(x) = \begin{cases} g(x), & x \in Y; \\ x, & x \in X \setminus Y. \end{cases}$$

   Prove that $i \colon \mathfrak{S}_Y \to \mathfrak{S}_X$ is an injective group homomorphism.
3. Let $f \colon X \to Y$ be a bijection. Prove that f induces a group isomorphism between $\mathfrak{S}_X$ and $\mathfrak{S}_Y$.

Aufgabe 4.

1. Let $\mathfrak{S}_n$ denote the set of bijections $\mathfrak{S}_X$ of a set $X = \{0, 1, \dots, n-1\}$. Prove that $\mathfrak{S}_n$ is not commutative for $n \geq 3$.
2. Construct an injective group homomorphism $\mathfrak{S}_n \times \mathfrak{S}_m \to \mathfrak{S}_{n+m}$.