LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Prof. Dr. Fabien Morel
Dr. Andrei Lavrenov, Oliver Hendrichs

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Lineare Algebra I – Tutoriumsblatt 02

Aufgabe 1.
Let $f\colon A \to B$ be a (set-theoretic) map.

1. Assume that f is injective. Prove that f defines a bijection between A and $f(A)$.
2. Prove that for a subset $C \subseteq B$ the following are equivalent:
   - $f^{-1}(C) = A$;
   - $f(A) \subseteq C$.

Aufgabe 2.
Consider maps $f\colon A \to B$ and $g\colon B \to C$.

1. Assume that f and g are injective (resp., surjective). Prove that $g \circ f$ is injective (resp., surjective).
2. Assume that $g \circ f$ is injective (resp., surjective). Prove that f is injective (resp., g is surjective).

Aufgabe 3.
A partition ρ of a set X is a set of non-empty subsets of X such that every element x in X is in exactly one of these subsets (i.e., the subsets are nonempty mutually disjoint sets). The sets in ρ are called its blocks (or parts, or cells).

The intersection $\rho \cap \rho'$ of two partitions ρ, ρ' of X is the set of all intersections of a block of ρ with a block of ρ', except for the empty sets.

Prove that the intersection of two partitions is again a partition.

Aufgabe 4.
For finite sets A, B prove that

$$|A \cup B| = |A| + |B| - |A \cap B|.$$