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## Lineare Algebra I – Tutoriumsblatt 13

### Aufgabe 1.

1. Find the roots of the polynomial  $x^2 - 2x + 2$  a) over  $\mathbb{C}$ ; b) over  $\mathbb{Z}/5\mathbb{Z}$ .
2. Find the roots of the polynomial  $x^3 + x^2 - 9x + 6$  over  $\mathbb{C}$ .
3. Decompose  $x^4 + 4$  as a product of indecomposable polynomials over  $\mathbb{R}$ .

### Aufgabe 2.

Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 5 & 6 & -3 \\ -1 & 0 & 1 \\ 1 & 2 & -1 \end{pmatrix}.$$

### Aufgabe 3.

Which of the following matrices are diagonalizable over  $\mathbb{C}$ ?

$$\begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ -2 & -2 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}.$$

### Aufgabe 4.

Let  $v_0 \in \mathbb{R}^2$ . Consider the recursively defined sequence of vectors  $(v_n)_{n \in \mathbb{N}}$  where

$$v_{n+1} = Av_n,$$

for  $A = \begin{pmatrix} 11 & -14 \\ 7 & -10 \end{pmatrix}$ . Determine all  $v_0$ , for which there exists  $N \in \mathbb{N}$ , such that for all  $n \geq N$  the first coordinate of  $v_n$  is greater than zero.