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Lineare Algebra I – Tutoriumsblatt 12

Aufgabe 1.
Compute the determinants of the following matrices:

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 4 & 0 & 3 & 1 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 & -1 & 2 \\ 1 & 1 & 2 & 0 \\ 4 & 0 & 2 & -1 \\ 3 & 2 & 0 & 4 \end{pmatrix}.$$

Aufgabe 2.
Find the inverse matrix for

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

1) using the cofactor matrix, and 2) using the Gaussian elimination.

Aufgabe 3.
Find the following determinants:

$$\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}, \qquad \begin{vmatrix} 2 & 1 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & \dots & 0 & 0 \\ 0 & 1 & 2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 2 \end{vmatrix}.$$

Aufgabe 4.
Let V be a finite-dimensional vector space over a field K, and $A\colon V \to V$ a non-zero linear map such that $A^2 = A$. Prove that there exists an eigenvector $v \in V$ with the eigenvalue 1.