LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Lineare Algebra I – Tutoriumsblatt 11

Aufgabe 1.
Compute the determinants of the following matrices:

1. $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$,
2. $\begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$,
3. $\begin{pmatrix} 1+\sqrt{2} & 2-\sqrt{5} \\ 2+\sqrt{5} & 1-\sqrt{2} \end{pmatrix}$,
4. $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$,
5. $\begin{pmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-1 & 0 & 1 \end{pmatrix}$,
6. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$, for $\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$.

Aufgabe 2.
Use the Laplace expansion to compute the determinant of the following matrix:

$$A = \begin{pmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 0 \\ -1 & 6 & 4 & 0 \end{pmatrix}.$$

Aufgabe 3.
Give an example of a subgroup of order 6 in $\mathfrak{S}_4$.

Aufgabe 4.
Consider a subset $\mathbb{H}$ of $\mathrm{M}_2(\mathbb{C})$ of all matrices of a form

$$\begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{pmatrix}.$$

1. Prove that $\mathbb{H}$ is a subring in $\mathrm{M}_2(\mathbb{C})$, which is also a $\mathbb{R}$-subspace of $\mathrm{M}_2(\mathbb{C})$.
2. Compute the determinant of a matrix from $\mathbb{H}$.
3. Prove that the set $\{a^2+b^2+c^2+d^2 \mid a,b,c,d \in \mathbb{Z}\}$ is closed under multiplication.