



Prof. Dr. Fabien Morel

Dr. Andrei Lavrenov, Oliver Hendrichs

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Lineare Algebra I – Tutoriumsblatt 10

Aufgabe 1.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \in M_2(K)$.

1. Find AB and BA .
2. Assume that $ad - bc \neq 0$. Find a matrix $C \in M_2(K)$ such that $AC = CA = E_2$.
3. Assume that $ad - bc = 0$. Prove that A is not invertible.

Aufgabe 2.

Solve the system of linear equations by transforming it into the row echelon form using Gaussian elimination.

$$\begin{cases} x_1 - 2x_2 + x_3 + 3x_4 = 1 \\ x_1 + x_2 + 2x_3 - x_4 = -1 \\ x_1 - 3x_2 + 4x_3 + 5x_4 = 5 \end{cases}$$

Aufgabe 3.

Let $L: U \rightarrow V$ be a linear map, and $P \leq Q \leq U$. Prove that $\dim L(Q) - \dim L(P) \leq \dim Q - \dim(P)$.

Aufgabe 4.

Show that any matrix $A \in M_{m,n}(K)$ of rank r can be presented as a product $A = BC$ for a matrix $B \in M_{m,r}(K)$ and $C \in M_{r,n}(K)$.