LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Prof. Dr. Fabien Morel
Dr. Andrei Lavrenov, Oliver Hendrichs

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Lineare Algebra I – Tutoriumsblatt 01

Aufgabe 1.
Let U be a set. For subsets A, $B \subseteq U$ denote

$$\overline{A} = U \setminus A \quad \text{and} \quad A \triangle B = (A \cap \overline{B}) \cup (\overline{A} \cap B).$$

Let A_i for $i \in I$ and A, B, C be subsets of U. Prove the following identities:

1. $\left(\bigcup_{i \in I} A_i\right) \cap B = \bigcup_{i \in I} (A_i \cap B)$;
2. $\left(\bigcap_{i \in I} A_i\right) \cup B = \bigcap_{i \in I} (A_i \cup B)$;
3. $\overline{\left(\bigcup_{i \in I} A_i\right)} = \bigcap_{i \in I} \overline{A}_i$;
4. $\overline{\left(\bigcap_{i \in I} A_i\right)} = \bigcup_{i \in I} \overline{A}_i$;
5. $A \triangle B = B \triangle A$;
6. $(A \triangle B) \triangle C = A \triangle (B \triangle C)$;
7. $A \triangle \emptyset = A$;
8. $A \triangle A = \emptyset$;
9. $(A \triangle B) \cap C = (A \cap C) \triangle (B \cap C)$;
10. $(A \triangle B) = (A \cup B) \setminus (A \cap B)$;
11. $(A \triangle B) = (A \setminus B) \cup (B \setminus A)$;
12. Prove that $(A \cap B) \subseteq C \Leftrightarrow A \subseteq \overline{B} \cup C$.

Aufgabe 2.
Let $f \colon X \to Y$ and $g \colon Y \to X$ be set-theoretic maps. We say that g is a *left* (corr., *right*) *inverse* of f if $g \circ f = \mathrm{id}_X$ (corr., $f \circ g = \mathrm{id}_Y$). Assume that X is non-empty. Prove that

1. f is injective if and only if it has a left inverse;
2. f is surjective if and only if it has a right inverse.

Aufgabe 3.
Let X be a set. Find a bijection between the set of all subsets of X and the set of all set-theoretic maps from X to the set $\{\emptyset, \{\emptyset\}\}$.

Aufgabe 4.
Prove that for any non-negative integer n, the integral part (entier) of $(\sqrt{3} + 2)^n$ is odd.