LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Summer term 2025 6. Mai 2025

Topology IV

Sheet 1

Exercise 1. Let $\pi\colon E \to B$ be a pointed spherical fibration, classified by a map $B \to \mathrm{Baut}_*(S^d)$. Consider the composite

$$\bar{\pi}\colon B \to \mathrm{Baut}_*(S^d) \subseteq \mathrm{An}_*.$$

Show that $\mathrm{Th}_*(\pi) \simeq \mathrm{colim}_B\, \bar{\pi}$.

Exercise 2. Let $\pi\colon E \to B$ be a pointed spherical fibration. Show that there is a canonical equivalence $\mathrm{Th}(\pi) \simeq \Sigma\mathrm{Th}_*(\pi)$.

Recall that $\mathrm{G}(d) = \mathrm{haut}(S^{d-1})$ and $\mathrm{F}(d) = \mathrm{haut}_*(S^d)$. There is then the forgetful map $\mathrm{F}(d) \to \mathrm{G}(d+1)$ and the suspension map $\mathrm{G}(d) \to \mathrm{F}(d)$. Recall also that a map f is called n-connected if its homotopy fibre is $(n-1)$-connected.

Exercise 3. Show that

1. the forgetful map $\mathrm{BF}(d) \to \mathrm{BG}(d+1)$ is d-connected, and
2. the composite $\mathrm{BF}(d) \to \mathrm{BG}(d+1) \to \mathrm{BF}(d+1)$ is d-connected.

In the exercises, maybe we can discuss also that the suspension map

$$\mathrm{BG}(d+1) \to \mathrm{BF}(d+1)$$

is in fact much better connected than what the above implies: it is roughly $2d$-connected.

This sheet will be discussed on May 9, 2025.