

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN



Summer term 2025

Algebraic *K*-theory

Sheet 0

Exercise 1. Let R be a ring and $x \in R$ an element. Show that the following are equivalent.

- 1. $x \in \operatorname{Jac}(R)$,
- 2. for all $y \in R$, we have that 1 xy is right-invertible,
- 3. Mx = 0 for all simple *R*-modules *M*.

Exercise 2. For an *R*-module *M*, define $Ann(M) = \{r \in R \mid Mr = 0\}$. Show that

$$\operatorname{Jac}(R) = \bigcap_{M \text{ simple}} \operatorname{Ann}(M).$$

Deduce that Jac(R) is a 2-sided ideal of R.

Exercise 3. Let R be a ring and $J \subseteq \operatorname{Jac}(R)$. Show that $\operatorname{Proj}^{\operatorname{fg}}(R) \to \operatorname{Proj}^{\operatorname{fg}}(R/J)$ is conservative.

Exercise 4. Let I be an ideal of a ring R such that R is I-adically complete. Show that idempotent elements of R/I can be lifted to idempotent elements of R.

This sheet will be discussed in the week of 23 October 2023.

5. Mai 2025