



Summer term 2025

May 20, 2025

# Topology IV

Sheet 4

**Exercise 1.** Show that  $\langle e(TS^n), [S^n] \rangle = 1 + (-1)^n = \chi(S^n)$ . Hint: Show that there is a canonical equivalence  $\text{Th}(TS^n) \simeq S^n \times S^n / \Delta$ , identify the Thom class, and compute its square.

**Exercise 2.** Compute  $H^*(\text{BSO}(d); \mathbb{F}_2)$  and  $H^*(\text{BSO}; \mathbb{F}_2)$ .

**Exercise 3.** Compute  $H^*(\text{BSp}(d); \mathbb{Z})$  and  $H^*(\text{BSp}; \mathbb{Z})$  in analogy to  $H^*(\text{BU}; \mathbb{Z})$  and  $H^*(\text{BU}(d); \mathbb{Z})$  and compute the map  $H^*(\text{BSp}; \mathbb{Z}) \rightarrow H^*(\text{BSp}; \mathbb{F}_2)$ . Compute the map induced on cohomology by  $\text{BU} \rightarrow \text{BSp}$  and that induced by  $\text{BSp} \rightarrow \text{BO}$  on  $\mathbb{Z}[\frac{1}{2}]$ -cohomology.

**Exercise 4.** Let  $X$  be a connected simple space all of whose homotopy groups are finite abelian groups. Show that  $H^*(X; \mathbb{Q}) = \mathbb{Q}$ .

This sheet will be discussed on May 30.