

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN



Summer term 2025

May 22, 2025

Algebraic *K*-theory

Sheet 3

Exercise 1. Show that the following condition on a Segal anima X are equivalent.

- (1) X is complete,
- (2) X^{\times} is a constant simplicial anima,
- (3) the diagram

$$\begin{array}{ccc} X_0 & \xrightarrow{s} & X_3 \\ & \downarrow \Delta & & \downarrow f \times g \\ X_0 \times X_0 & \xrightarrow{s \times s} & X_1 \times X_1 \end{array}$$

is a pullback, where f and g are induced by the two maps $[1] \rightarrow [3]$ in Δ given by $0 \mapsto 0, 1 \mapsto 2$ and $0 \mapsto 1, 1 \mapsto 3$, respectively.

(4) The map $X_0 \to \operatorname{Map}_{sAn}(J, X)$ induced by $J \to *$ is an equivalence. Here, J denotes the nerve of the contractible groupoid with two elements.

Exercise 2. Let \mathcal{C} be an ∞ -category. Show that its Rezk nerve $N(\mathcal{C})$ is a complete Segal anima.

Exercise 3. Let $M \in CMon(An)$. Characterize when its underlying simplicial anima is complete.

Exercise 4. Let \mathcal{C} be an ∞ -category with small limits. Show that $CMon(\mathcal{C})$ and $Mon(\mathcal{C})$ again have small limits and that the forgetful functors

$$\operatorname{CMon}(\mathcal{C}) \to \operatorname{Mon}(\mathcal{C}) \to \mathcal{C}$$

preserve small limits.

This sheet will be discussed in the week of 23 October 2023.