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Symmetries of surfaces: The mapping class groups

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The topology of surfaces is an area that carries surprising and unexpected complexity. Despite the fact that connected surfaces are easily classified (as connected sums of tori and projective planes), the symmetries or self-maps of surfaces carry a much richer structure. It is captured in the mapping class group of the surface: the group of self-homeomorphisms of the surface viewed up to continuous deformation.
Our goals for the seminar will be to gain understanding of the following insights about the mapping class groups:

- The mapping class group is finitely generated.
- The mapping class group acts on the singular homology of the surfaces. This gives a group representation, i.e., a homomorphism to a group of integer matrices, whose image is given by the symplectic matrices.
- Each homeomorphism of the surface defines an automorphism of its fundamental group, unique up to conjugation. This induces a homomorphism from the extended mapping class group to the outer automorphism group of the fundamental group, which is in fact an isomorphism.

Intended Audience: Master's students in Mathematics, TMP students, interested Bachelor's students (who are not afraid of a challenge)

Prerequisites: We will assume point-set topology, and homology. Many arguments use fundamental groups and covering spaces – ideally these are known, but the required results will be briefly recalled in the beginning of the seminar. Some knowledge of Differential Geometry (in particular hyperbolic surfaces) is also helpful, but will also be recalled.

Literature:

- Benson Farb and Dan Margalit, "A Primer on Mapping Class Groups"
- John Stillwell, "Classical Topology and Combinatorial Group Theory"
- Putman, "A quick proof of the classification of surfaces" (available here `https://www3.nd.edu/~andyp/notes/ClassificationSurfaces.pdf`)

Time/Form: The seminar will be offered as a *block seminar* in the week October 9 to October 13. During this week we will meet every day (alternating between LMU and TU), with several talks per day. This will allow us to immerse us in the topic and hopefully understand the material deeply.

List of talks (preliminary):

1. **Mapping Class Group of the Torus**: straight line representatives for simple closed curves, characterising intersection number via determinants, matrix description of mapping classes, examples [Stillwell 6.4, Primer 2.2.4]
2. **Hyperbolic Surfaces**: Hyperbolic plane as universal cover of genus g surfaces, examples, quick reminder of lifting properties for covering spaces. Endpoints at infinity for loops, Intersections via linking. [Primer 1.2]
3. **Classification of Surfaces**: Classification of surfaces [Putman, Farb-Margalit]
4. **Curve Facts**: Geometric intersection number, bigon criterion, change of Coordinates Principle [Primer 1.2, 1.3]
5. **Mapping Class Examples**: Torsion elements, Dehn twists and their basic properties, Pseudo-Anosov examples via branched covers.
6. **Finite Generation I (Removing Punctures)**: Short exact sequence for replacing boundaries with punctures, Birman Exact Sequence [Primer 3.6, 4.2]
7. **Finite Generation II (Curve Graphs)**: The Curve Graph is connected and has finite quotient [Primer 4.1]
8. **Finite Generation III (Induction)**: A geometric group theory lemma, and combining everything [Primer 4.3]
9. **Symplectic Representation I (Homology and simple closed curves)**: The "Euclidean algorithm for curves", action of Dehn twists [Primer 6.2, 6.3.1]
10. **Symplectic Representation II (Surjectivity)**: Various points of view on surjectivity, noninjectivity [Primer 6.3.2]
11. **Symplectic Representation III (Torsion)**: Torelli is torsion free [Primer 6.4.2,3]
12. **Mapping Classes via fundamental groups**: The Dehn-Nielsen-Baer theorem [Primer 8]