Tutorial 12

Let k be a field.

1. Let $U \subset \mathbb{A}^1_k$ be the complement of two distinct closed points in $\mathbb{A}^1_k$. Consider the sheaf $\mathcal{F}$ on $\mathbb{A}^1_k$ which is defined on an open subset $V \subset \mathbb{A}^1_k$ by

$$\mathcal{F}(V) = \begin{cases} k & \text{if } \emptyset \neq V \subset U, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find a flasque resolution

$$0 \to \mathcal{F} \to \mathcal{G} \to \mathcal{H}_1 \oplus \mathcal{H}_2 \to 0$$

where $\mathcal{G}, \mathcal{H}_1, \mathcal{H}_2$ are appropriate skyscraper sheaves.

b) Deduce from your resolution that $H^1(\mathbb{A}^1_k, \mathcal{F}) \neq 0$. Why is this not a contradiction to $\mathbb{A}^1_k$ being affine?

2. Using Čech cohomology, compute the dimension of $H^i(\mathbb{P}^2_k, \mathcal{O}_{\mathbb{P}^2_k}(-5))$ for all $i \geq 0$.

3. Let $X = \mathbb{A}^2_k \setminus \{(0,0)\}$.

a) Let $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module.

Show that $H^i(X, \mathcal{M}) = 0$ for all $i > 1$.

b) Let $\mathcal{K}_X$ be the constant sheaf with values in the function field of X.

Show that $H^i(X, \mathcal{K}_X/\mathcal{O}_X) = 0$ for all $i > 0$.