# Tutorial 10

Let $k$ be a field.

**1.** Consider the ring

$$A = \left( \frac{k[x,y]}{(xy)} \right)_{(x,y)} .$$

Recall from Tutorial 2 that

$$X := \operatorname{Spec} A = \{(x), (y), (x,y)\}$$

and

$$\mathcal{O}_X(\{(x)\}) = k(y) \qquad \mathcal{O}_X(\{(y)\}) = k(x) \qquad \mathcal{O}_X(\{(x),(y)\}) = k(x) \times k(y) .$$

The $A$-linear map $A \to A$ given by $a \mapsto ax$ induces a homomorphism $f \colon \mathcal{O}_X \to \mathcal{O}_X$ of $\mathcal{O}_X$-modules.

Show that $\mathcal{M} = \ker f$ and $\mathcal{N} = \operatorname{coker} f$ are coherent $\mathcal{O}_X$-modules and explicitly compute $\mathcal{M}(U)$ and $\mathcal{N}(U)$ for the three open subsets $U \subset X$ from above.

**2.** Consider the morphism $f \colon \mathbb{P}^1_k \to \mathbb{P}^2_k$ which is given in homogeneous coordinates by

$$[x : y] \mapsto [x^5 : x^2y^3 : y^5] .$$

Compute the pullback $f^*\mathcal{O}_{\mathbb{P}^2_k}(1)$.

**3.** True or false? Find a proof or a counterexample.

| **Statement** | **True** | **False** |
|---|---|---|
| If $X$ is an integral scheme with function field $K$, the constant sheaf with values in $K$ is a quasi-coherent $\mathcal{O}_X$-module. | ☐ | ☐ |
| If $i \colon U \to X$ is an open immersion, the pushforward $i_*\mathcal{O}_U$ is a coherent $\mathcal{O}_X$-module. | ☐ | ☐ |
| Any coherent $\mathcal{O}_X$-module on $X = \mathbb{A}^2_k \setminus \{(0,0)\}$ is of the form $\mathcal{M}\vert_X$ for a coherent $\mathcal{O}_{\mathbb{A}^2_k}$-module $\mathcal{M}$. | ☐ | ☐ |