Tutorial 9

Let k be a field.

1. Which of the following morphisms of k-schemes are proper?

    a) $\operatorname{Spec} k[x,y]/(x^2, y^3) \to \operatorname{Spec} k$

    b) $\mathbb{A}^1_k \setminus \{-1\} \to \mathbb{A}^2_k$ given on k-rational points by $t \mapsto (t^2, t^3 - t)$

    c) $\mathbb{P}^2_k \setminus \{p\} \to \operatorname{Spec} k$ where $p \in \mathbb{P}^2_k$ is a closed point

2. Show that a proper morphism between affine schemes is finite.

3. True or false? Find a proof or a counterexample.

Statement	True	False
A bijective morphism of schemes is closed.	☐	☐
There exists a proper morphism $\mathbb{A}^2_k \to \mathbb{A}^1_k$.	☐	☐
The composition of two projective morphisms is projective.	☐	☐