Tutorial 8

1. Compute the following tensor products.

$$\mathbb{Z}/6 \otimes_{\mathbb{Z}/60} \mathbb{Z}/10 \qquad \mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q} \qquad \mathbb{C}[x]/(x^2) \otimes_{\mathbb{R}} \mathbb{C}[x]/(x^2) \qquad \mathbb{R}(x) \otimes_{\mathbb{R}[x]} \mathbb{C}[x]$$

2. Let X and Y be schemes defined over fields of different characteristic. Show that the fibre product $X \times_{\mathbb{Z}} Y$ is empty.

3. Let $X \to S$ be a morphism of schemes. Show that the diagonal morphism

$$\Delta \colon X \to X \times_S X$$

is separated.

4. Consider the $\mathbb{R}$-scheme

$$X = \operatorname{Proj} \frac{\mathbb{R}[x, y, z]}{(x^2 + y^2 + z^2)} .$$

Show that X is not isomorphic to $\mathbb{P}^1_{\mathbb{R}}$, but the $\mathbb{C}$-scheme

$$X_{\mathbb{C}} = X \times_{\mathbb{R}} \mathbb{C}$$

is isomorphic to $\mathbb{P}^1_{\mathbb{C}}$.

5. True or false? Find a proof or a counterexample.

Statement	**True**	**False**
For any fibre product $X \times_S Y$ of schemes, the projection morphism $X \times_S Y \to X$ is separated.	□	□
If $f \colon X \to Y$ is a surjective morphism of schemes such that Y is connected and the fibre $f^{-1}(y)$ is connected for all $y \in Y$, then X is connected.	□	□
A finite morphism of schemes is always separated.	□	□