Tutorial 6

1. Decide for each of the following rings A whether the morphism $\operatorname{Spec} A \to \operatorname{Spec} \mathbb{Z}$ is finite or of finite type, respectively.

$$\mathbb{Z}[\sqrt{3}] \qquad \mathbb{Z}[\tfrac{1}{3}] \qquad \mathbb{Z}_{(3)} \qquad \mathbb{Z}/(3) \qquad \mathbb{Z} \times \mathbb{Z} \qquad \mathbb{Z}[x]$$

2. Let X be a quasi-compact scheme. Show that the closure of every point $x \in X$ contains a closed point.

3. Let X be a noetherian scheme. Show that any morphism $X \to Y$ which is locally of finite type is of finite type.

4. Let X be a scheme. Show that a point $x \in X$ is closed if and only if the corresponding morphism $\operatorname{Spec} \kappa(x) \to X$ is finite.