Tutorial 5

Let k be a field.

1. Decide for each of the following schemes whether they are connected, irreducible, reduced, or integral.

$$\operatorname{Spec} \mathbb{Z}/9 \qquad \operatorname{Spec} \mathbb{Z}/10 \qquad \operatorname{Spec} \mathbb{Z}/11 \qquad \operatorname{Spec} \mathbb{Z}/12$$

2. Which of the following schemes are isomorphic to each other?

$$\operatorname{Spec} k \qquad \operatorname{Spec}(k \times k) \qquad \operatorname{Proj} k[x] \qquad \operatorname{Spec} k[x]_x$$

$$\operatorname{Proj}(k[x,y]) \setminus V(xy) \qquad \operatorname{Proj} \frac{k[x,y]}{(xy)}$$

(We assume $\deg x = \deg y = 1$ in all cases.)

3. Consider the graded ring $S = k[u,v]$ where $\deg u = 2$ and $\deg v = 3$. Show that the scheme $\operatorname{Proj} S$ is isomorphic to $\mathbb{P}^1_k$.

4. True or false? Find a proof or a counterexample.

Statement	**True**	**False**
A scheme is irreducible if and only if it has a generic point.	☐	☐
A morphism $f\colon X \to Y$ of schemes over k sends a k-rational point of X to a k-rational point of Y.	☐	☐
Let X be a scheme. If $\mathcal{O}_X(X)$ is reduced, then X is reduced.	☐	☐
A dense open subset of a connected scheme is connected.	☐	☐
If X is an integral scheme, then for any non-empty open subset $\emptyset \neq U \subset X$ the restriction map $\mathcal{O}_X(X) \to \mathcal{O}_X(U)$ is injective.	☐	☐