Tutorial 4

1. Let $(X, \mathcal{O}_X)$ be a locally ringed space and let $f \in \mathcal{O}_X(X)$ be a global section. Show that the zero set

$$\{x \in X \mid f(x) = 0 \text{ in } \kappa(x) := \mathcal{O}_{X,x}/\mathfrak{m}_{X,x}\}$$

is closed in X.

2. True or false? Find a proof or a counterexample.

Statement	**True**	**False**
If X is a scheme and $U, V \subset X$ are affine open subsets, then $U \cap V$ is an affine open subset.	☐	☐
Let $f\colon X \to Y$ be a morphism of schemes. If $f^\#\colon \mathcal{O}_Y \to f_*\mathcal{O}_X$ is an isomorphism, then f is an isomorphism.	☐	☐
Every scheme contains a dense affine open subset.	☐	☐
If X is a scheme whose underlying topological space is a finite set, then X is affine.	☐	☐
If two global sections $f, g \in \mathcal{O}_X(X)$ of a scheme X agree on a dense open subset $U \subset X$, then $f = g$.	☐	☐

3. Let k be a field. Show that any morphism of schemes from $\mathbb{P}^1_k$ to an affine scheme is constant.

4. Let $X = \operatorname{Proj} S$ for a graded ring S. Show that for any two points $x, y \in X$, there exists an affine open subset $U \subset X$ such that $x, y \in U$.